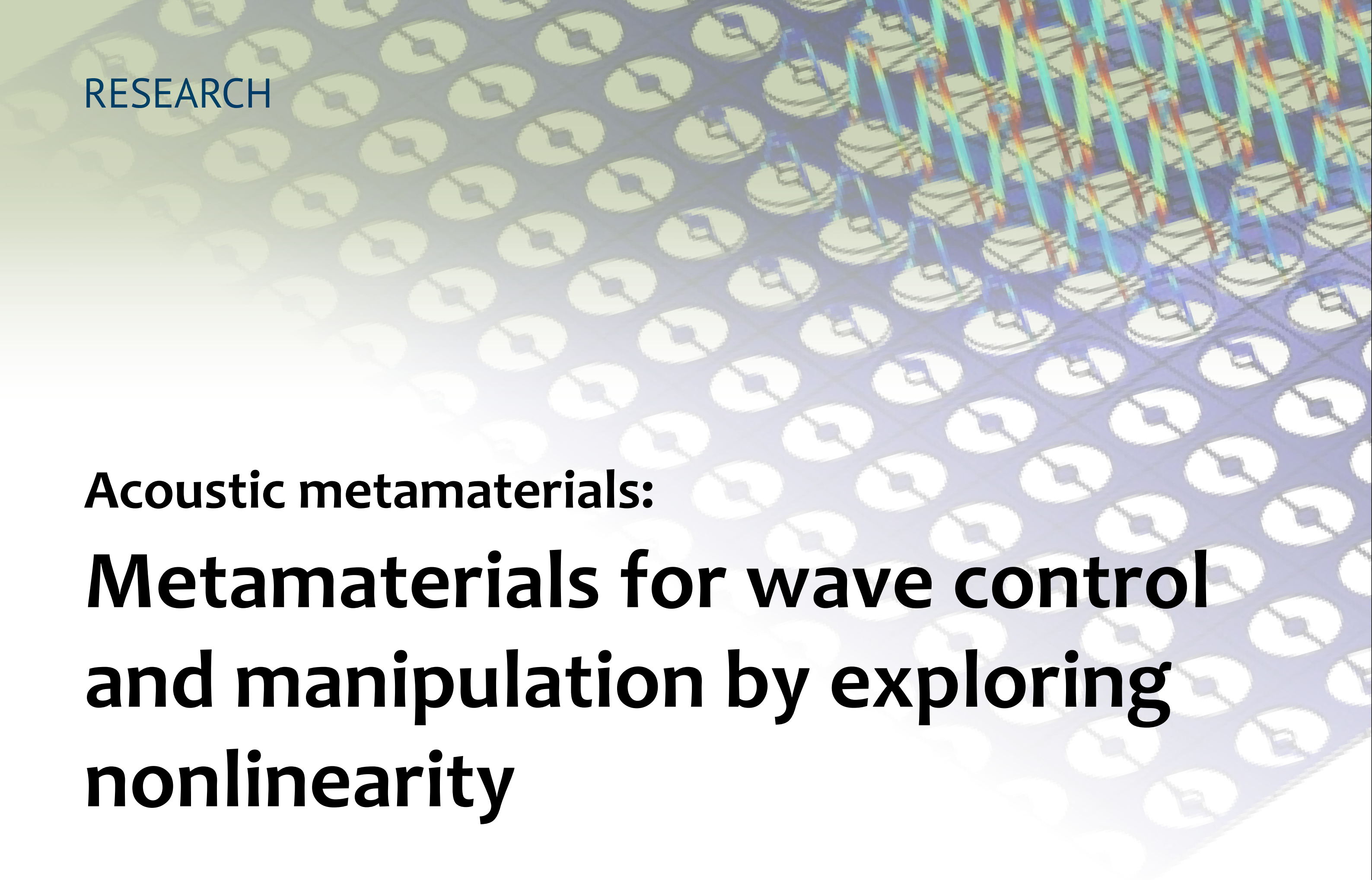

Acoustic metamaterials:

Metamaterials for wave control and manipulation by exploring nonlinearity

The development of metamaterials enables to engineer materials with extraordinary features, beyond the traditional limits. In the linear dynamic regime, metamaterials have already enabled a wide range of new functionalities, such as cloaking, super-lenses, and signal filtering. The consideration of nonlinearity has the potential to bring a myriad of new opportunities for metamaterials. Within the 4TU.High-Tech Materials research program, metamaterials were developed with nonlinear resonant inclusions. Results of this research show emergent dynamic features which may enable tunability, new mechanisms of sound and vibration attenuation, and the realization of a 'mechanical diode'. Besides, efficient computational schemes are being developed for optimal analysis of the emergent metastructure design.

Would it be possible to manipulate acoustic and elastic waves in a rational manner, by providing tunable control of wave propagation and attenuation, and enabling new functionalities as, for example, logic and sequential operations or complex quantum computation? The answer is yes, and the recent developments on metamaterials prove this is possible. Metamaterials are engineered structures in which the design of a microscopic structure, or meta-atom, with specific behavior (for example dynamics) gives rise to superb, on-demand response of the effective medium, beyond that of its constituents, making it possible to manipulate waves. Indeed, wave manipulation is promoted by the unusual dispersion characteristics of metamaterials, that is (i.e.) the relation between frequency and wavenumber of free waves propagating in a medium, induced by 'exotic' effective properties, such as negative effective density and/or elastic moduli. In the dispersion diagram of metamaterials, these features may induce frequency zones in which waves cannot propagate, the so-called band gaps, and negative slopes, i.e. negative group velocity with positive phase velocity, enabling unusual reflection, transmission and absorption characteristics within the band gaps, and unique refraction characteristics within certain pass bands.

In the early 90s, the success of photonic crystals for electromagnetic wave manipulation motivated the development of phononic crystals, i.e. periodic structures with contrasting materials and/or geometries inducing band gaps by destructive wave interference, the so-called Bragg scattering phenomenon. The work by Liu et al. (1) was strikingly important, demonstrating that localized resonances can promote subwavelength band gaps, i.e. in the regime of wavelengths orders of magnitude larger than the unit cell dimensions. Indeed, Liu's material was the first realization of the so-called locally resonant metamaterial exhibiting negative effective density. A scheme depicting the typical dispersion diagram and transmission plot for Liu's

material considering 1D periodicity and longitudinal wave propagation only is shown in Figure 1. Acoustic metamaterials with negative effective elastic moduli have been realized a few years later using an array of Helmholtz's resonators, thus opening the possibility of realizing negative refractive index elastoacoustic materials, used to design focusing acoustic lenses and cloaking devices. Besides, a wide range of functionalities has been proposed for acoustic metamaterials, such as: filtering, phase manipulation, and absorption.
All the above-mentioned studies and related functionalities have been designed considering linear dynamic regime only. What would happen if materials are pushed into a nonlinear regime? In the past, nonlinear static and dynamic regimes in mechanical structures were avoided by design due to the complex behaviors induced by nonlinearity. The advent of metamaterials is creating a new paradigm in which nonlinear phenomena are regarded as an opportunity for enabling new and extraordinary functionalities. For instance, snap-through instability mechanism induced by buckling has been used to conceive soft actuators, which would respond with large outputs to small inputs (2). Bistable structures have been used to release energy in a controlled way, enabling one-way propagation of solitary transition waves over arbitrary long distances (3). Higher-order harmonics generation due to wave propagation in a nonlinear medium has been used to induce mode-conversion (4) and unusual reflection effects (5).

Within the 4TU.High-Tech Materials research program, high-performance locally resonant metamaterials are being developed by exploring nonlinearities within the internal resonant microstructures. So far, few studies have investigated the effect of nonlinearity in resonant attachments periodically distributed in a host material. Moreover, conventional strategies to simulate the dynamic behavior of such nonlinear metamaterials are either computationally expensive (for example transient direct numerical simulations) or restricted to simple models (for example method of multiple scales, harmonic balance method). Thus, the aim of the 4TU.High-Tech Materials research project is two-fold: (i) unravel physical phenomena induced by nonlinearities within the microstructure of locally resonant metamaterials and (ii) develop efficient computational schemes for their analysis.

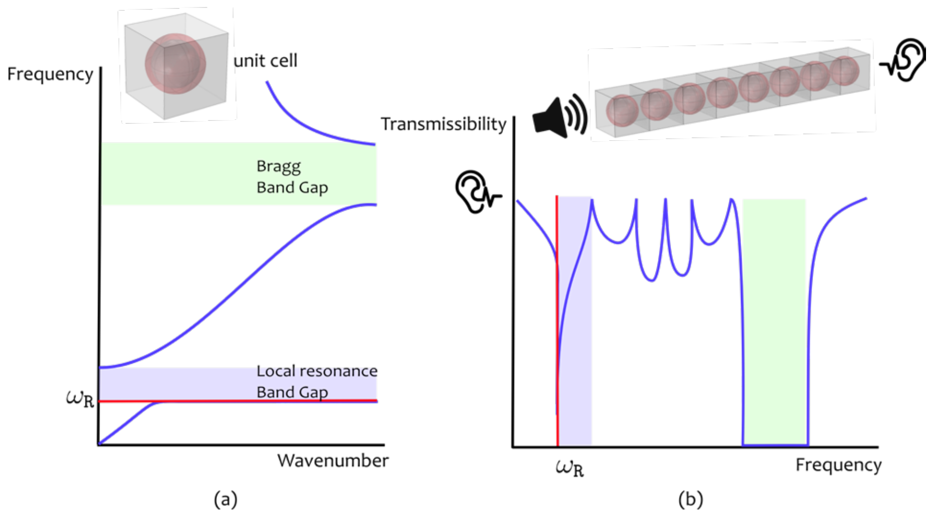

Figure 1: Basic features of metamaterials (1D periodicity and longitudinal wave propagation): (a) dispersion diagram based on the unit cell analysis, (b) corresponding transmissibility plot (ω_R is the local resonance frequency)

Emergent dynamic features

Hyperelastic and viscoelastic materials are widely used in structural connections functioning as elastic springs and/or dash-pots. Since the first design of a locally resonant metamaterial, rubber coatings have been used to provide the compliance required to make the heavy inclusion resonate inside the host medium. However, in that work and in numerous works inspired by the emergent local resonance phenomenon using rubber as coupling microstructural element, the analysis has typically been restricted to the linear regime of small oscillations. Within the framework of the 4TU.High-Tech Materials research project, the focus is on the nonlinear dynamic regime of hyperelastic materials, characterized by their asymmetric tension-compression material behavior.

Tunability

Nonlinear material behavior of rubber-like materials is characterized by strain-dependent elastic modulus, for example described by neo-Hookean material model, illustrated in Figure 2. This induces an effective amplitude-dependent frequency shift of acoustic and optical wave modes, and consequently,

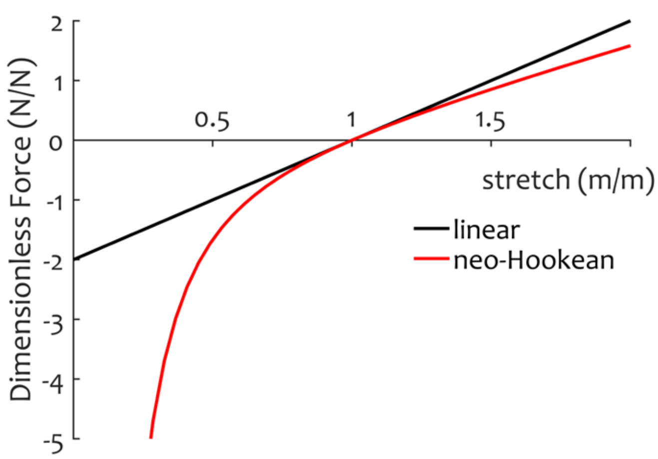

Figure 2: Linear and nonlinear (neo-Hookean) local interaction forces versus stretch in locally resonant metamaterials

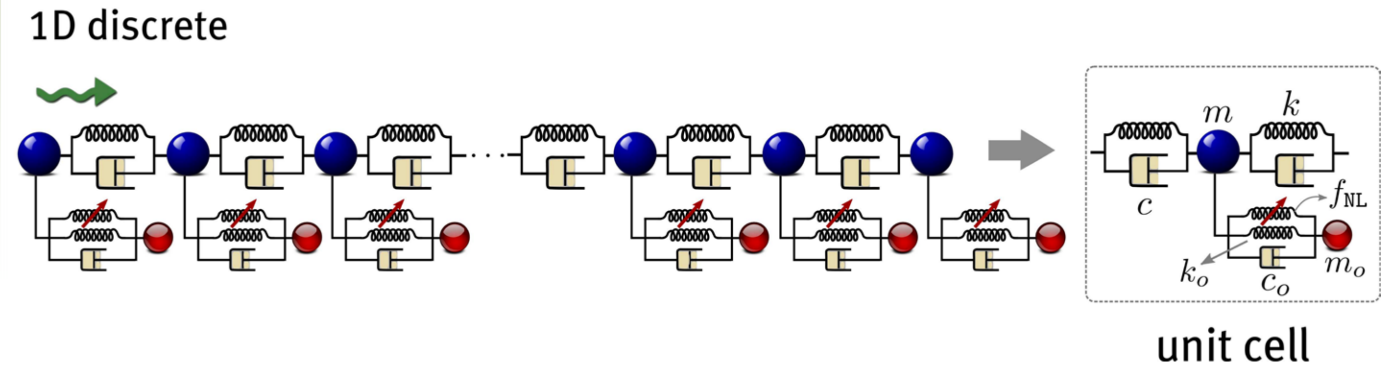

Figure 3: One-dimensional model of metamaterial with nonlinear local resonators

of the band gap. This feature of nonlinear materials makes nonlinear phononic crystals and locally resonant metamaterials tunable since transmission/reflection features of these materials can be modified by changing the input amplitude. Numerical wave propagation analysis of a one-dimensional lo-

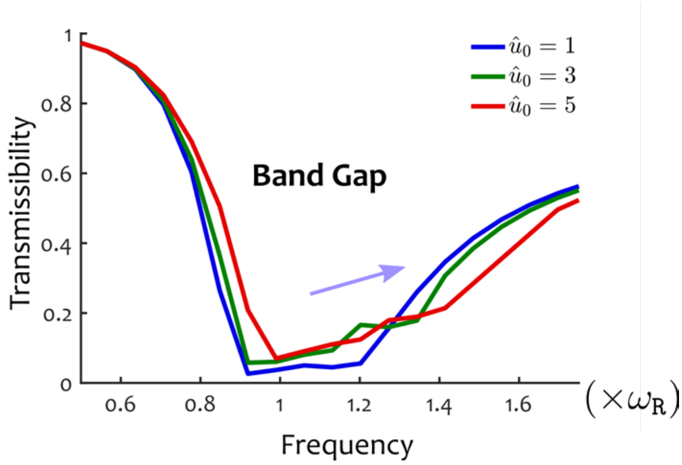

Figure 4: Amplitude-dependent transmission of a nonlinear locally resonant metamaterial: onset of tunability

cally resonant metamaterial (Figure 3) with local interaction given by an incompressible neo-Hookean material model shows a frequency shift of the local resonance band gap towards high frequencies with the increase of the input wave amplitude (see Figure 4). Although asymmetric, i.e. undergoing softening

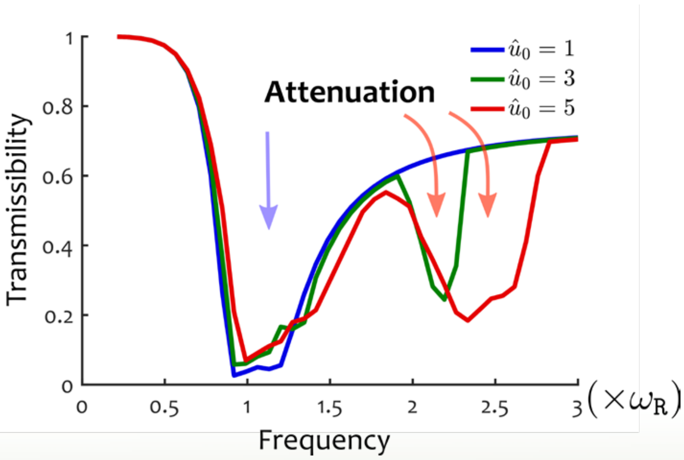

Figure 5: Transmission analysis of a nonlinear locally resonance metamaterial depicting a second attenuation zone

in tension and stiffening in compression, the stiffening effect is stronger and explains the feature of these elastomeric metamaterials.

Parametric attenuation

In the dynamic analysis of the metamaterial with nonlinear and asymmetric local interaction of neo-Hookean type, a second attenuation zone (or transmission deep) was observed for sufficiently high excitation amplitudes (see Figure 5). Semi-analytical analysis using the method of multiple scales has been performed and showed that the energy exchange between the primary propagating wave mode and a subharmonic

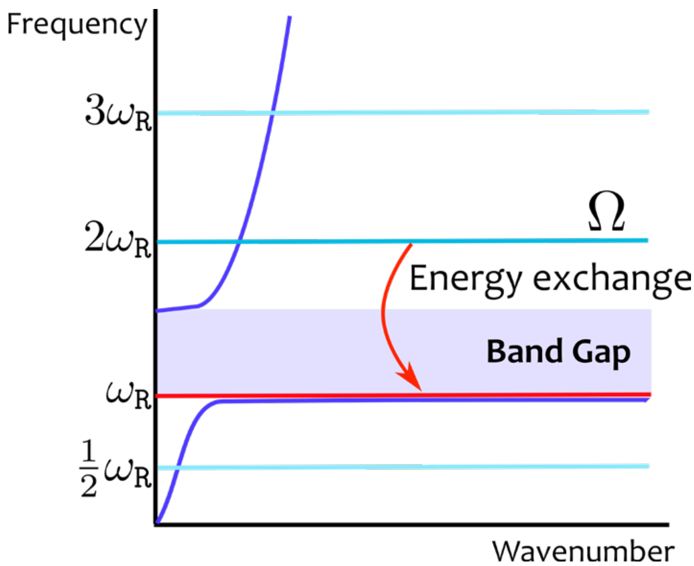

Figure 6: Scheme of the mechanism responsible for the second attenuation zone depicting energy exchange due to autoparametric resonance

evanescent mode might occur above a certain critical excitation amplitude. This explains the emergent phenomenon, which is due to the so-called autoparametric resonance at the microstructure level (6). Indeed, above a certain energy level, the primary propagating wave mode with frequency-wavenumber pair (Ω,μ) becomes unstable and energy is transferred to the evanescent wave mode at the one-half subharmonic of the primary input frequency (1⁄2)Ω (see Figure 6), inducing the oscillator response at its resonance frequency. The nature of this phenomenon suggests that similar attenuation zones

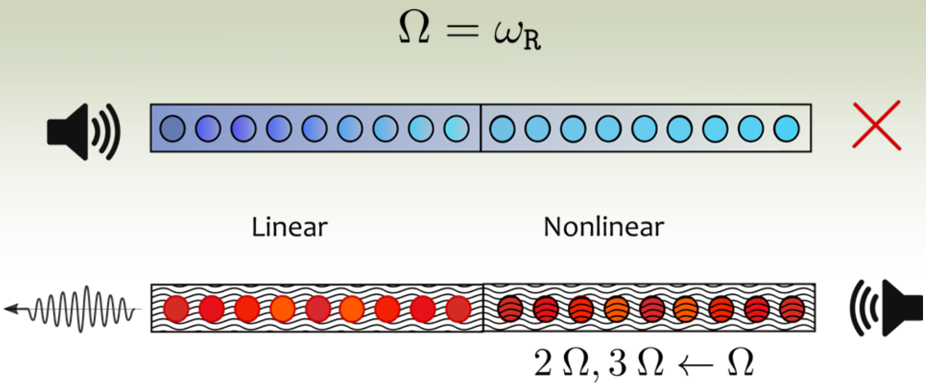

Figure 7: Nonreciprocal behavior of a structural system composed of linear and nonlinear locally resonant metamaterials connected in series and excited with frequency $\Omega=\omega_R$

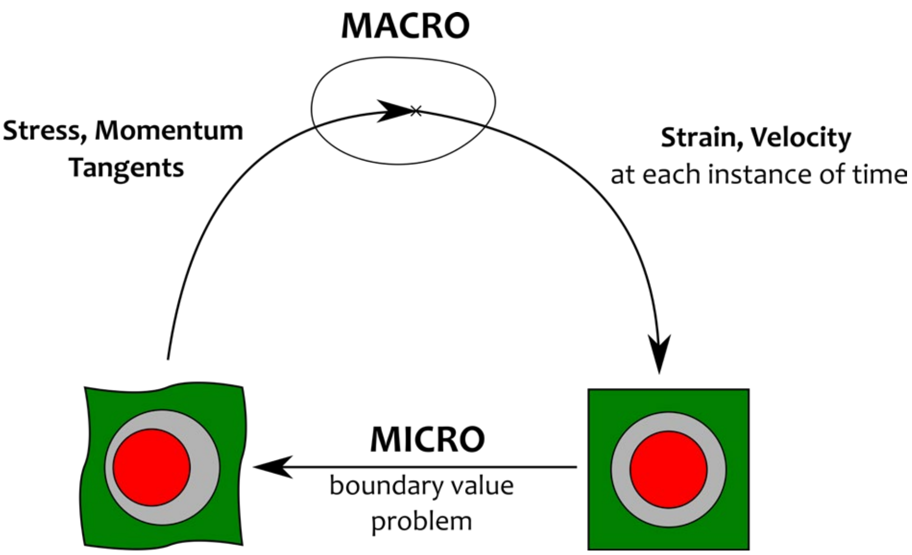

Figure 8: Scheme of dynamic computational homogenization implemented to perform transient analysis of nonlinear metamaterial

are expected to be induced at specific conditions, not only for a 2:1 frequency ratio. This novel mechanism of inducing attenuation by the interplay between nonlinearity and local resonance is inherently tunable and offers new avenues for wave manipulation in the subwavelength regime.

Nonreciprocity

The incorporation of nonlinearities constitutes a way of designing materials through which wave transmission is direction dependent (nonreciprocal materials), offering novel possibilities for sound and vibration control. Within the framework of this 4TU.High-Tech Materials research project, a structure composed of two distinct metamaterial waveguides has been designed, as shown in Figure 7. A metamaterial with linear resonant inclusions (on the left) is coupled in series to a metamaterial with nonlinear resonant inclusions (on the right). The designed mechanical system works as a mechanical diode, providing a nonreciprocal response around a particular frequency $\Omega=\omega_R$. If the excitation source is on the left, no wave is transmitted to the right end. On the contrary, if the source is on the right, a wave signal is propagated to the left, as a result of harmonic generation in the nonlinear metamaterial. This structural system could be used in solutions for signal processing, imaging, sensor technology, among others.

Transient analysis via computational homogenization

Besides the emergent features induced by nonlinearity in metamaterials, efficient ways to perform transient wave propagation analysis on finite size structures is needed. Homogenization methods, i.e. a class of multiscale methods based on averaging theorems, are powerful tools in the analysis of composite materials and can be applied to the locally resonant metamaterials design for two main reasons.

Firstly, the extraordinary features of metamaterials emerge from the effective properties of the composite material rather than its constituents. Secondly, the main feature of the locally resonant metamaterials is their subwavelength feature which makes them suitable for homogenization. Recently, both analytical and computational homogenization schemes have been extended to incorporate micro-inertia and allow complex transient interactions. When complex microstructures need to be modeled in detail, computational homogenization schemes can be used and are typically more efficient than the conventional direct numerical simulations.

In 2013, Pham et al. (7) extended the classical computational homogenization scheme to incorporate micro-dynamics. However, the numerical implementation provided at that time was restricted to a linear elastic locally resonant metamaterial only. Within the framework of the 4TU.High-Tech Materials research program, the dynamic computational homogenization scheme incorporating microscopic nonlinearities, schematically shown in Figure 8, was implemented and transient numerical simulations were performed (8). For the validation, a one-dimensional version of Liu's locally resonant metamaterial was chosen and the dynamics of the nonlinear metamaterial was assessed under free wave propagation and transient structural dynamic analysis of a finite structure (see Figure 9). The results showed to be accurate, while the simulation time is significantly smaller compared to the conventional direct numerical simulations.

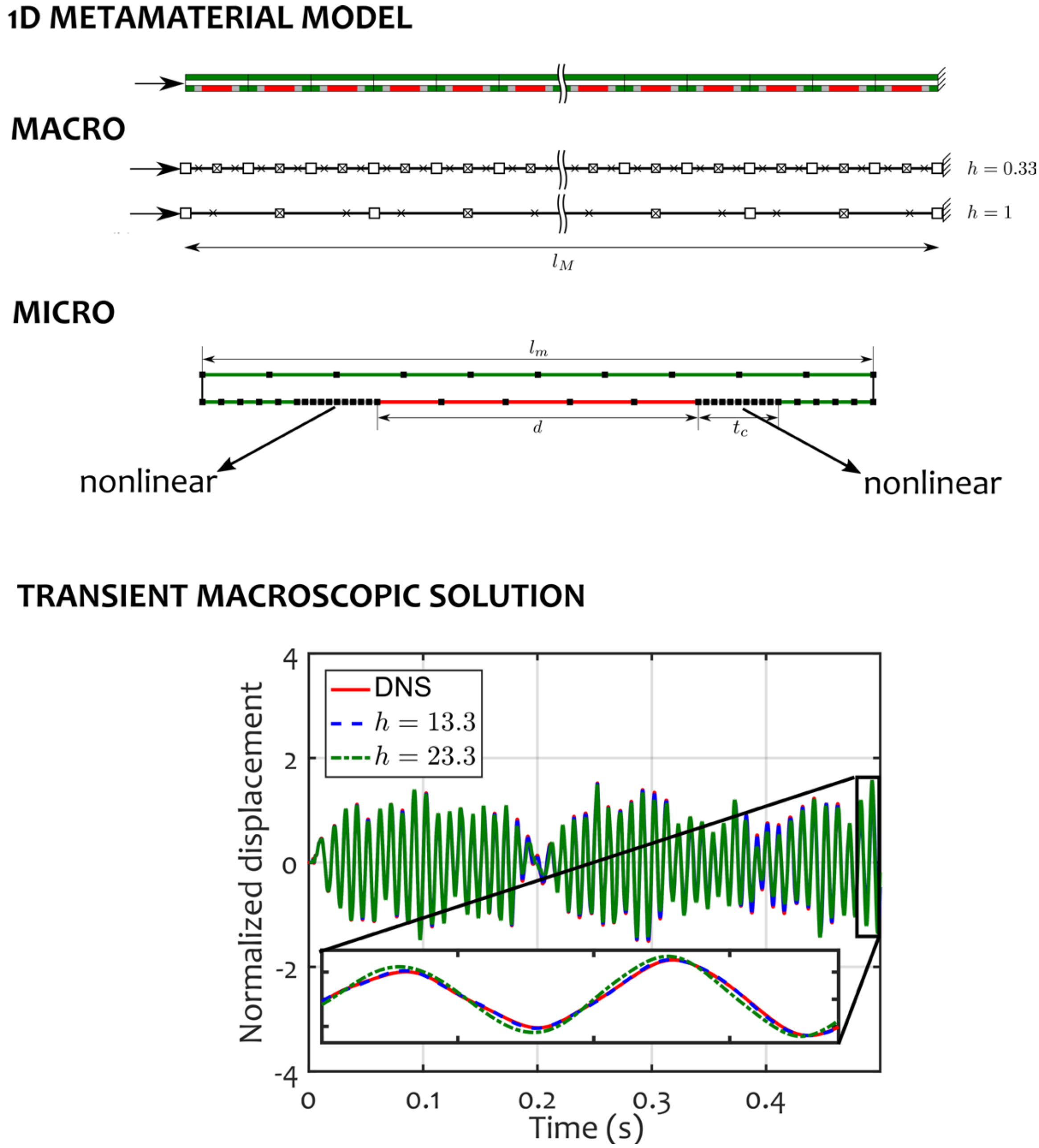

Figure 9: 1D nonlinear metamaterial transient problem analyzed via computational homogenization and direct numerical simulations. Full problem, macro and micro scales are shown. A typical solution is depicted for two levels of the amount of homogenization parameter h in comparison with solution via the conventional direct numerical simulation (DNS), with good agreement observed (8)

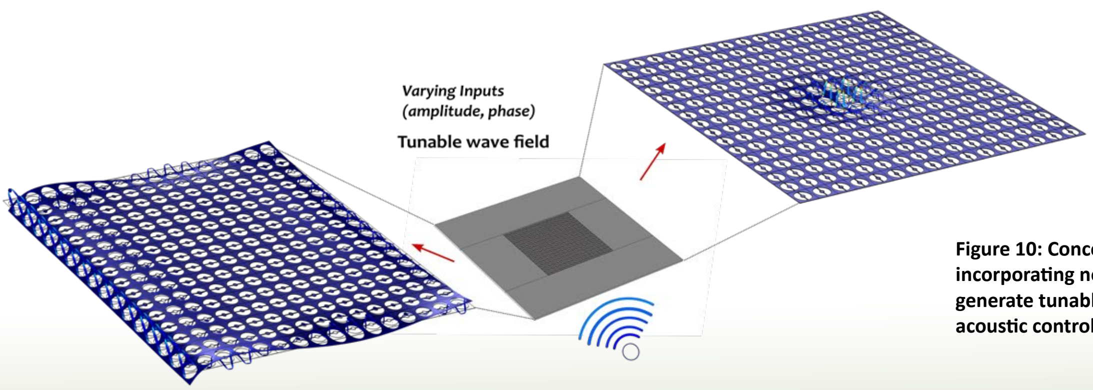

Figure 10: Conceptual scheme of a device incorporating nonlinear metamaterial able to generate tunable wave field for vibration and acoustic control

Towards an emergent metamaterial design

Design of a locally resonant metamaterial subwavelength wave manipulation exhibiting the novel features induced by local nonlinearities for a real application in vibration/acoustic signal filtering, as shown in Figure 10, is the ultimate goal of this 4TU.High-Tech Materials research project. To this end, the first mechanical prototype is being conceived (Figure 11) and will be numerically and experimentally tested. In this first prototype, the nonlinear effect responsible for parametric attenuation will be induced by geometrical nonlinearity within the local resonator. This will provide a proof of the concept and will allow the application of the computational homogenization scheme to a complex nonlinear metamaterial model.

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https://www.4tu.nl/htm/en/new-horizons/metamaterials/

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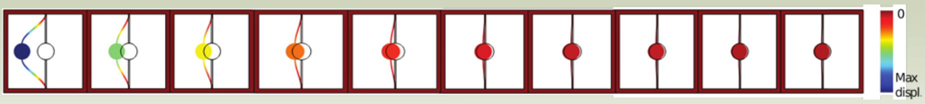

Figure 11: Mechanical prototype of a nonlinear metamaterial able to provide parametric attenuation

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