Enriched finite element modelling of interface problems

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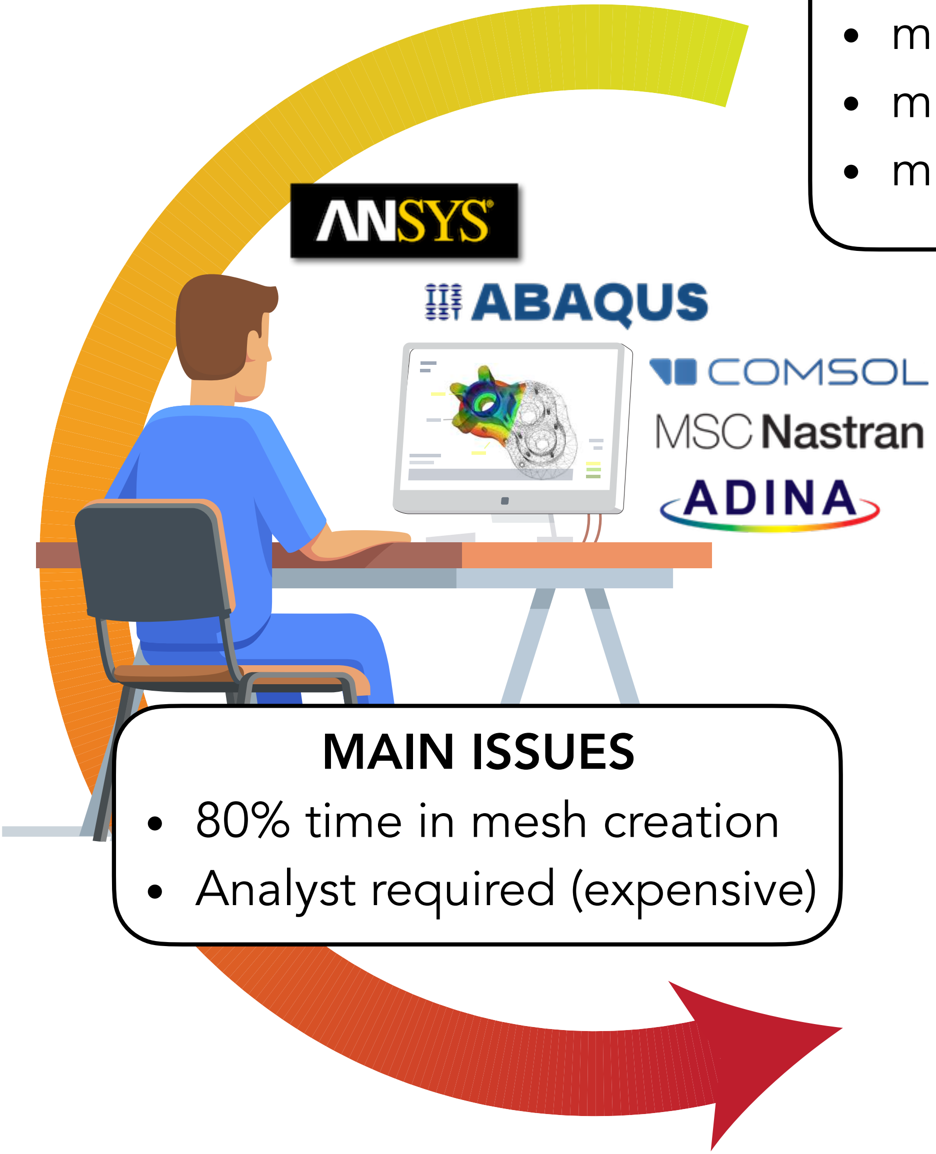
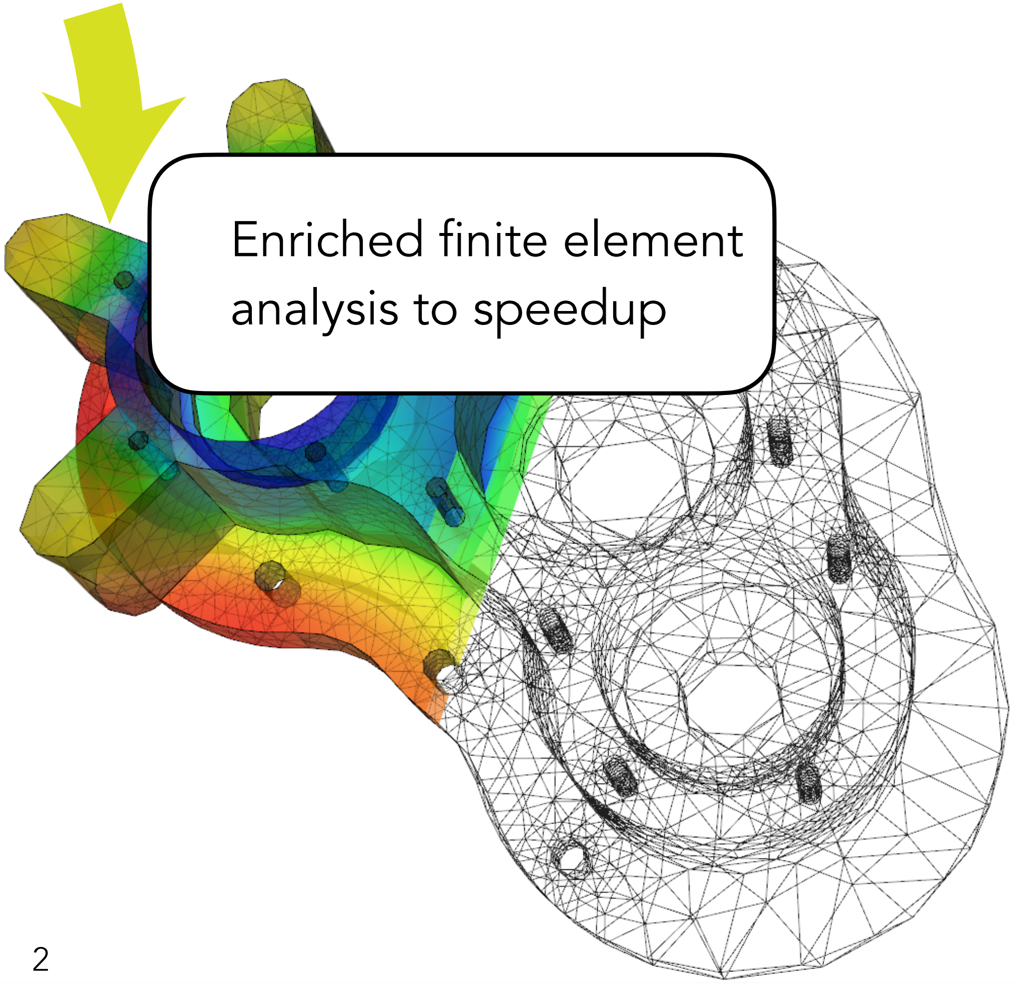
EXAMPLE: COMPLEX PART

- multiple materials
- multiple interfaces
- multiple meshes



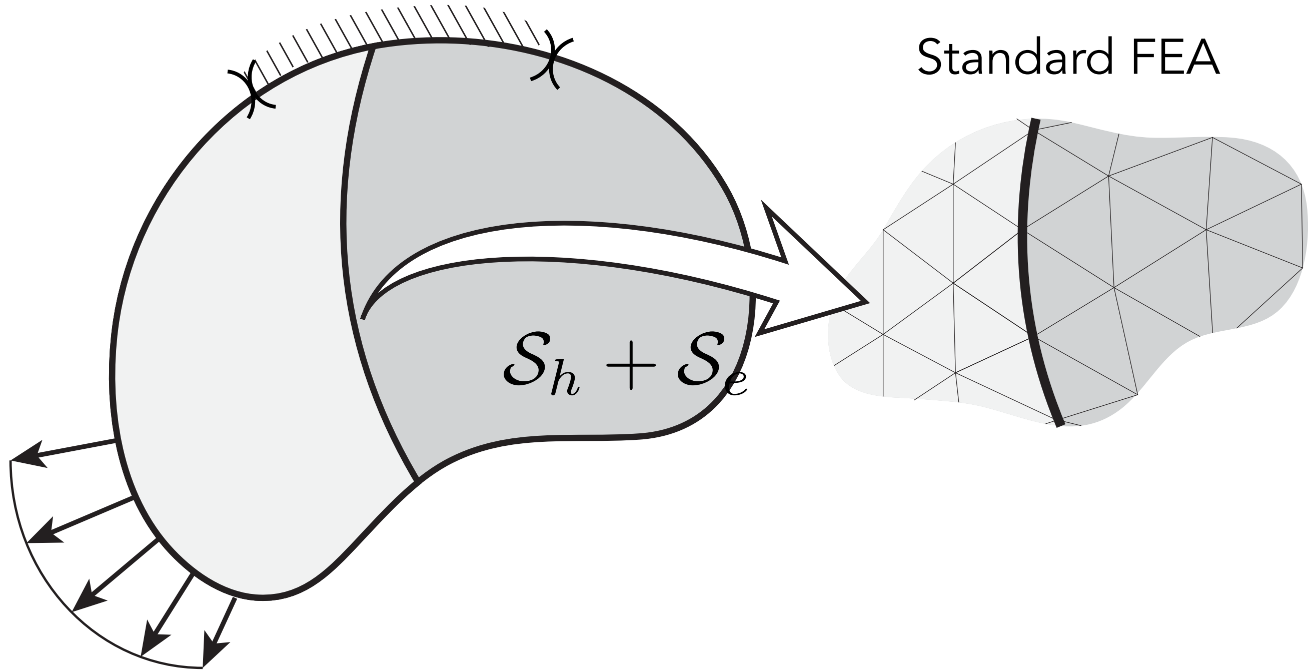
MAIN ISSUES

- 80% time in mesh creation
- Analyst required (expensive)

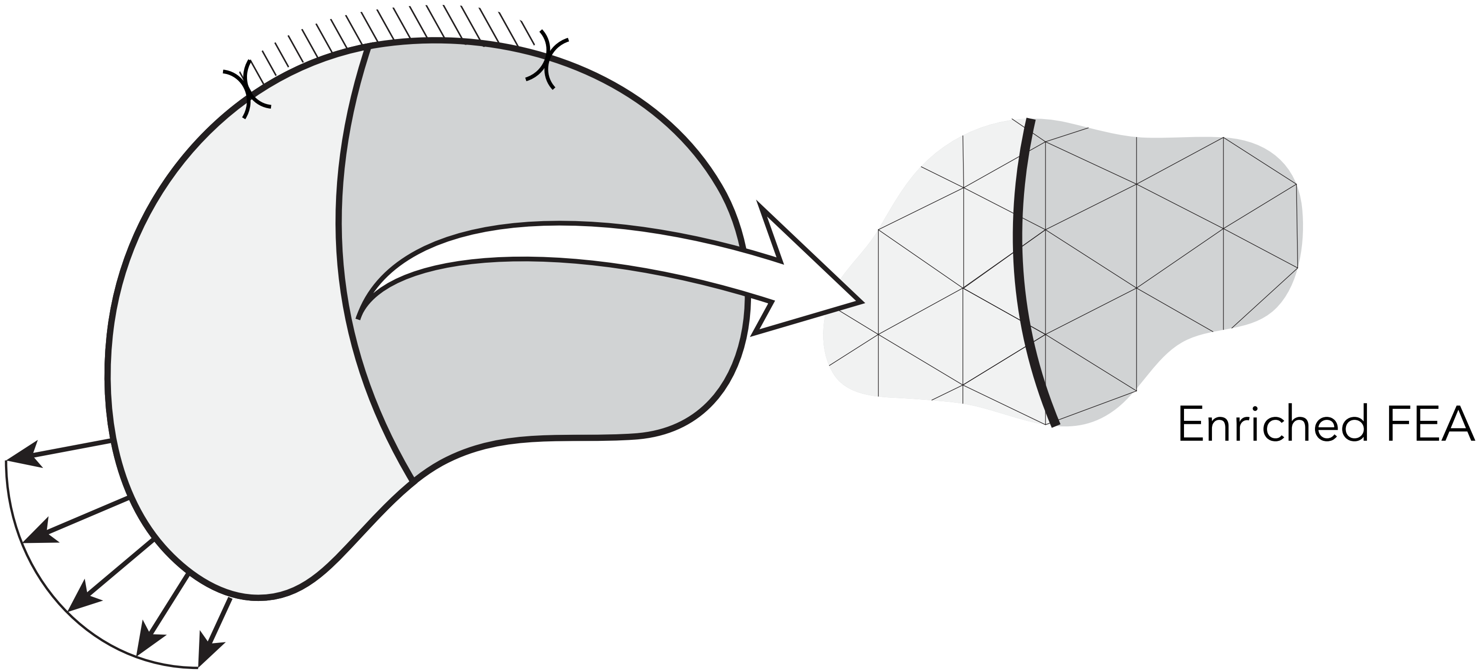


Enriched finite element analysis to speedup

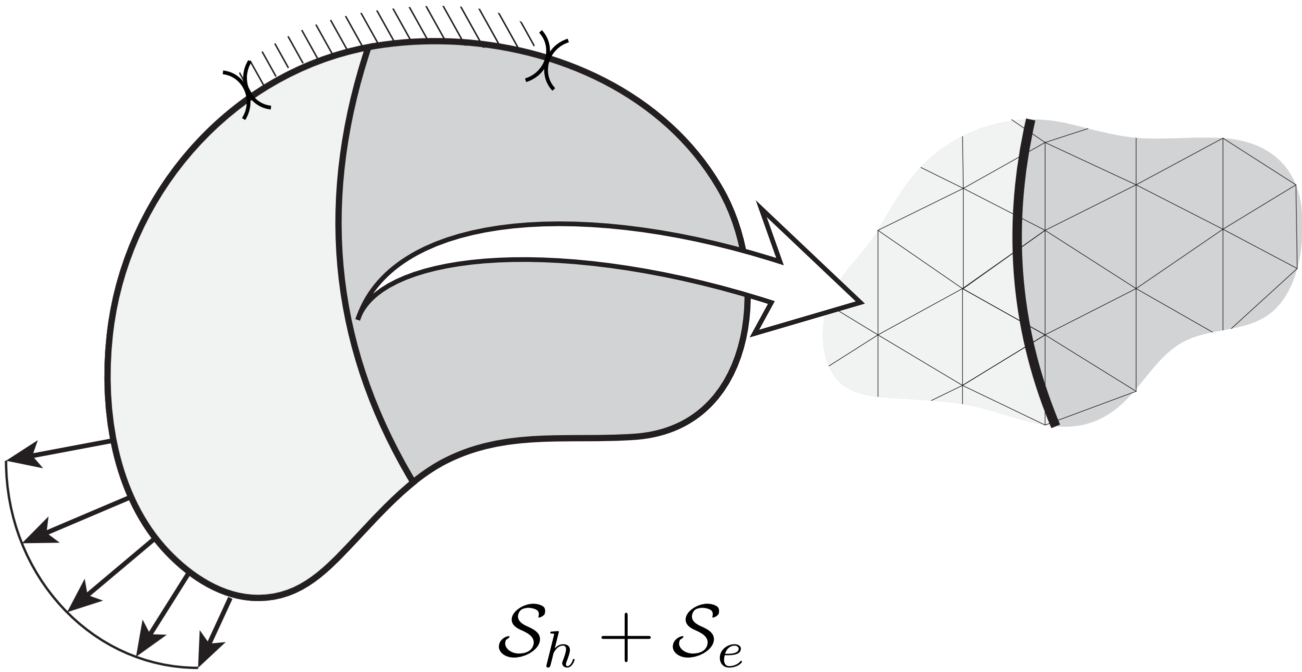
Enriched finite element methods allow us to decouple geometry from discretization



Enriched finite element methods allow us to decouple geometry from discretization



Enriched finite element methods allow us to decouple geometry from discretization



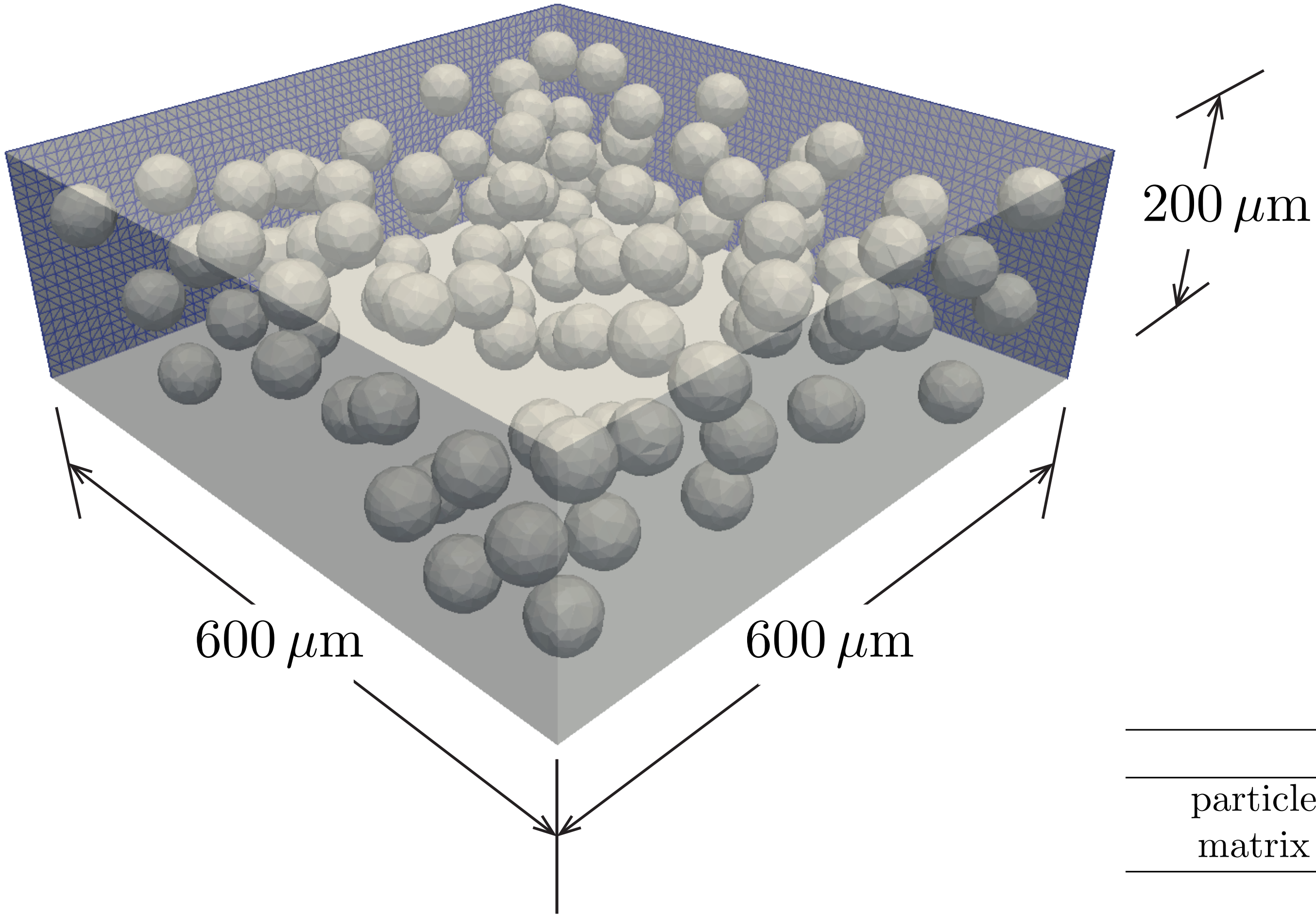
$$\mathcal{S}_h + \mathcal{S}_e$$

$$u^h(\boldsymbol{x}) = \underbrace{\sum_{i \in \iota_h} N_i(\boldsymbol{x})\, u_i}_{\text{standard FEM}} + \underbrace{\sum_{i \in \iota_e} s_i \psi_i(\boldsymbol{x})\, \alpha_i}_{\text{enriched}}$$

- Soghrati *et al.*, An interface-enriched generalized FEM for problems with discontinuous gradient fields. *Int J Numer Meth Eng*, 89 (2012)

Decoupling mesh from geometry makes it easy to obtain statistically significant results

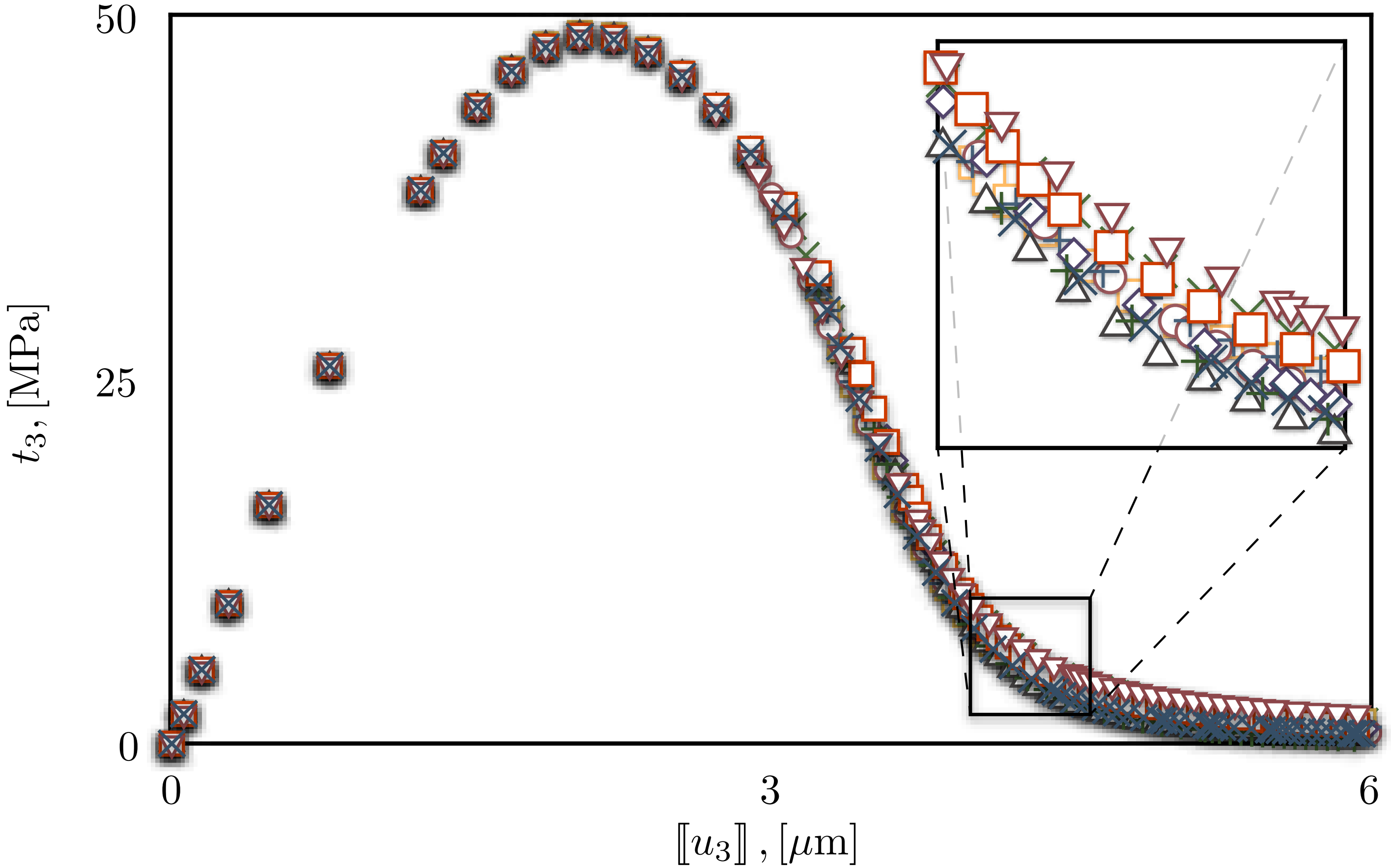
- 10% volume of elastic spherical inclusions (110 total each with 50 μm in diameter)



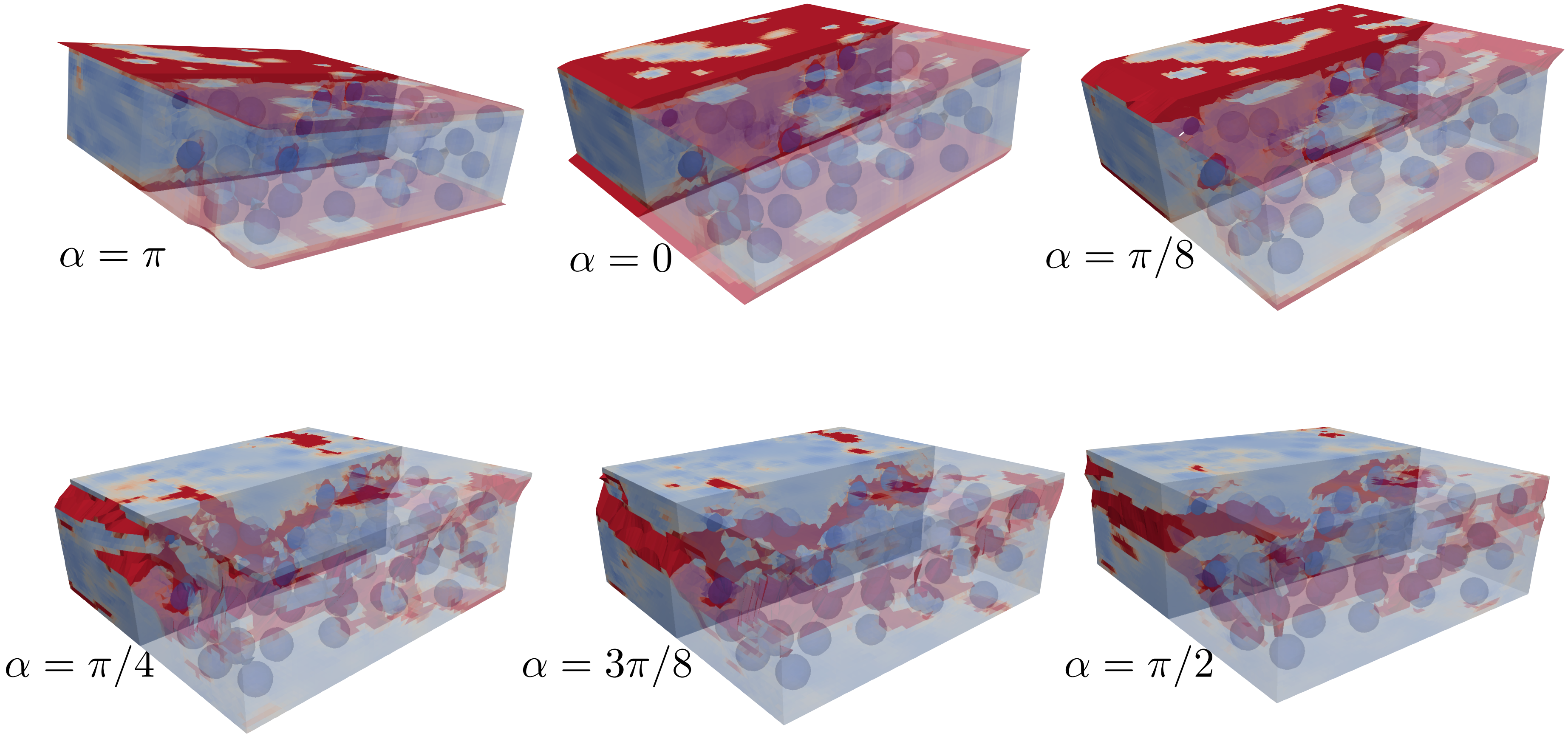
	E, [GPa]	ν
particle	20	0.3
matrix	3.9	0.34

Decoupling mesh from geometry makes it easy to obtain statistically significant results

- 10% volume of elastic spherical inclusions
  (110 total each with 50 µm in diameter)



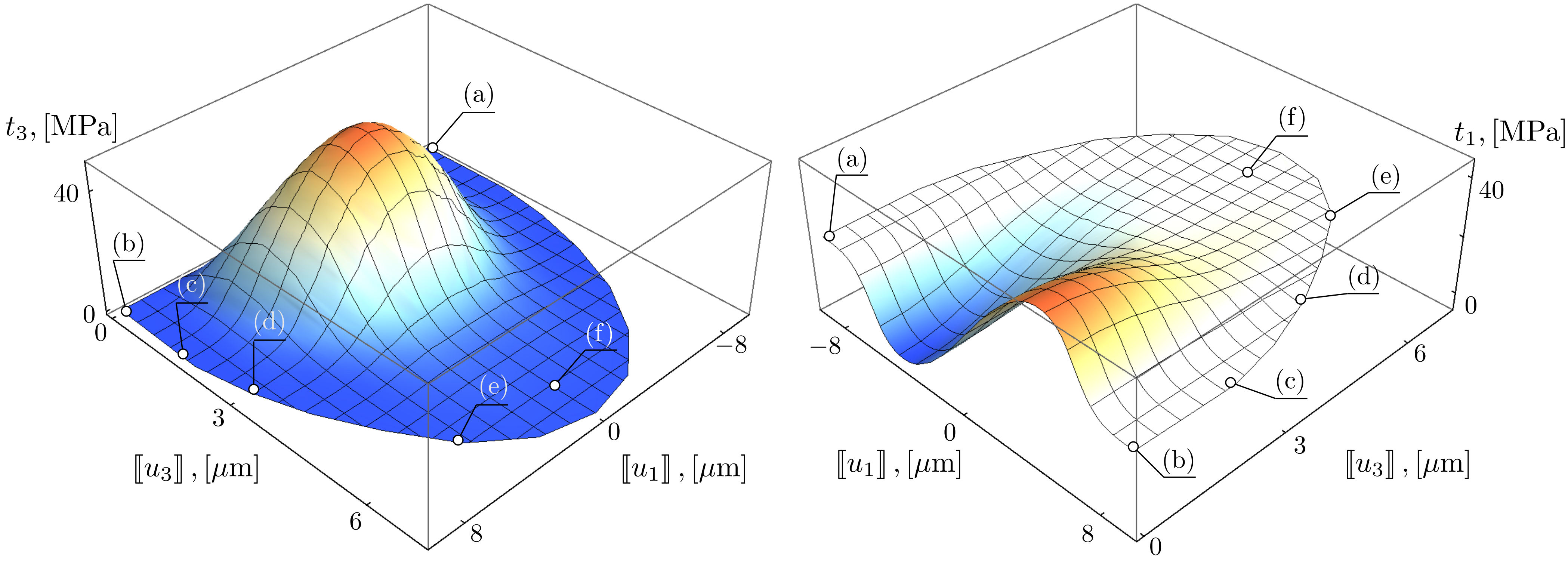
The formulation was used to obtain failure envelopes



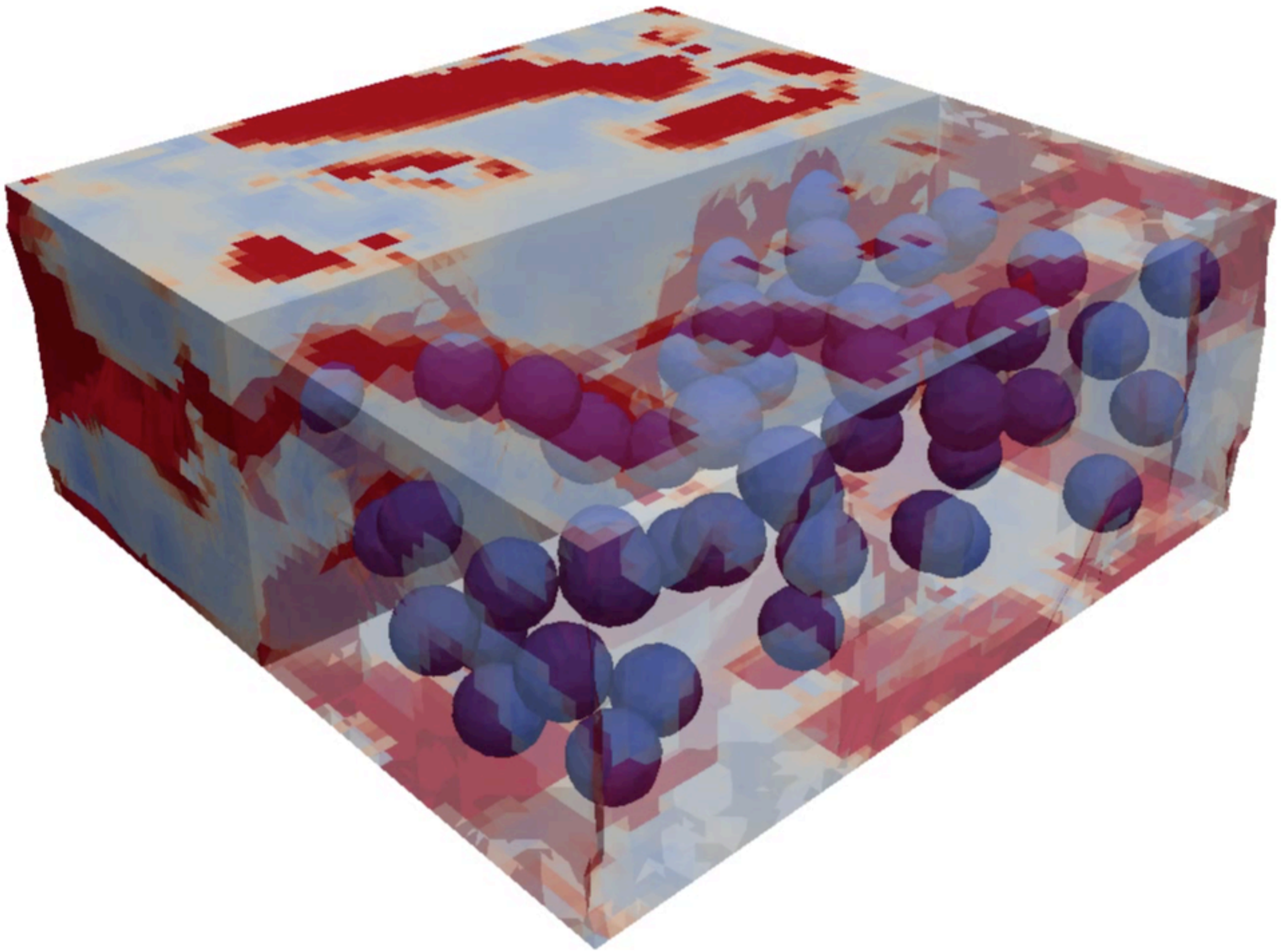
The formulation was used to obtain failure envelopes

- Macroscopic displacement jump:

$$[\![u_1(\tau)]\!] = 0.4\,\tau\cos(\alpha)\,,$$
$$[\![u_3(\tau)]\!] = 0.2\,\tau\sin(\alpha)\,.$$



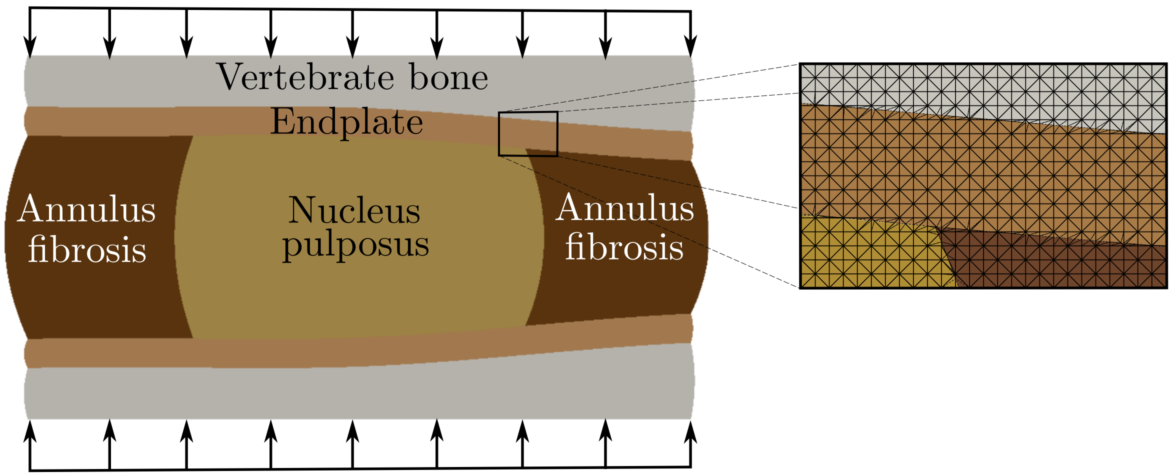
The effect of in-plane deformation is captured in the failure response

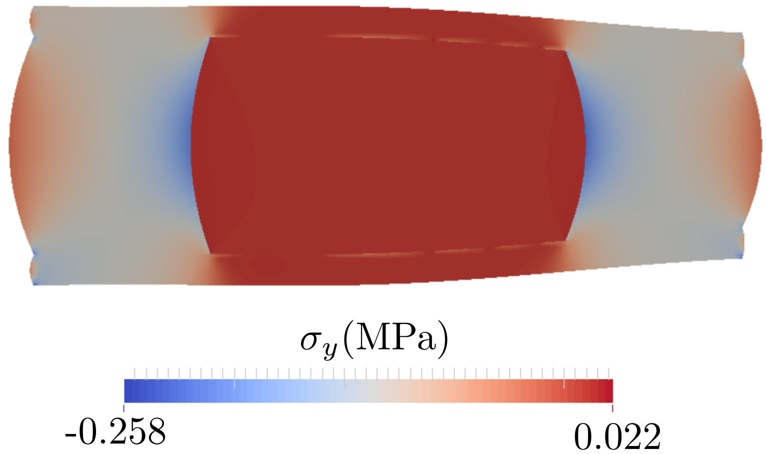


- Aragón *et al.*, Effect of in-plane deformation on the cohesive failure of heterogeneous adhesives. *J Mech Phys Solids*, 61 (2013)

Enriched FEM can accurately capture the stress response of an intervertebral disc

- Human intervertebral disc (IVD):
  - 200×100 structured mesh;
  - Condition number 7.82×10^9 to 3.45×10^3 (same as standard FEM).

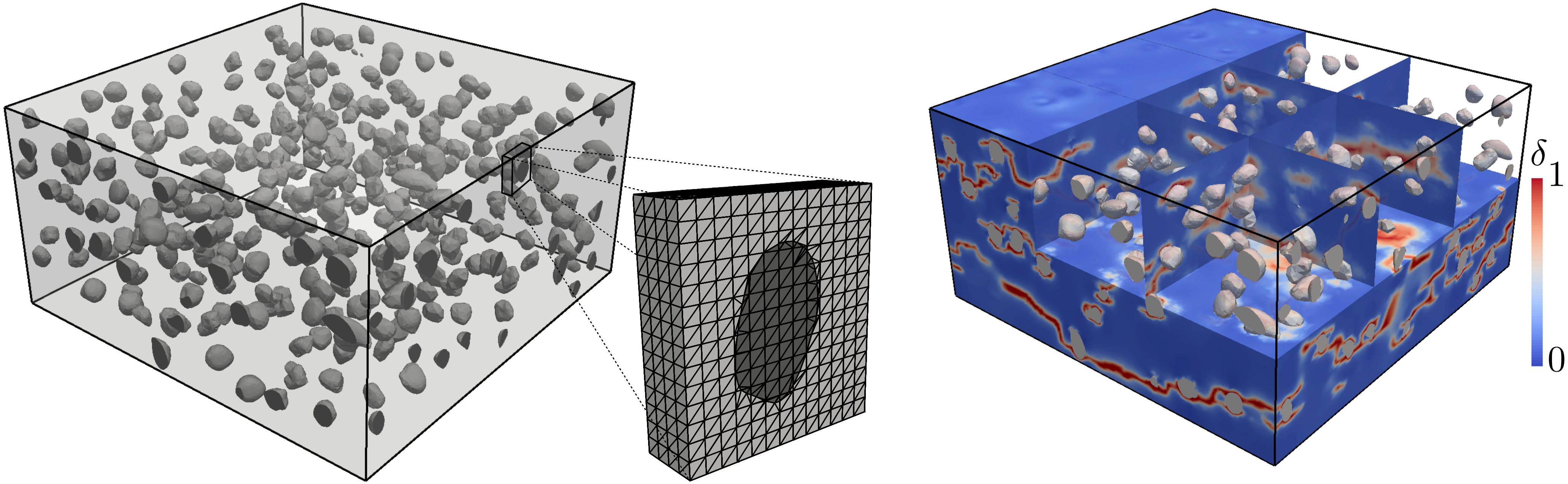




- Aragón *et al.*, On the stability and interpolating properties of the Hierarchical Interface-enriched Finite Element Method. *Comput Methods Appl Mech Eng*, 362 (2020)

The interface-enriched formulation is stable with respect to condition number

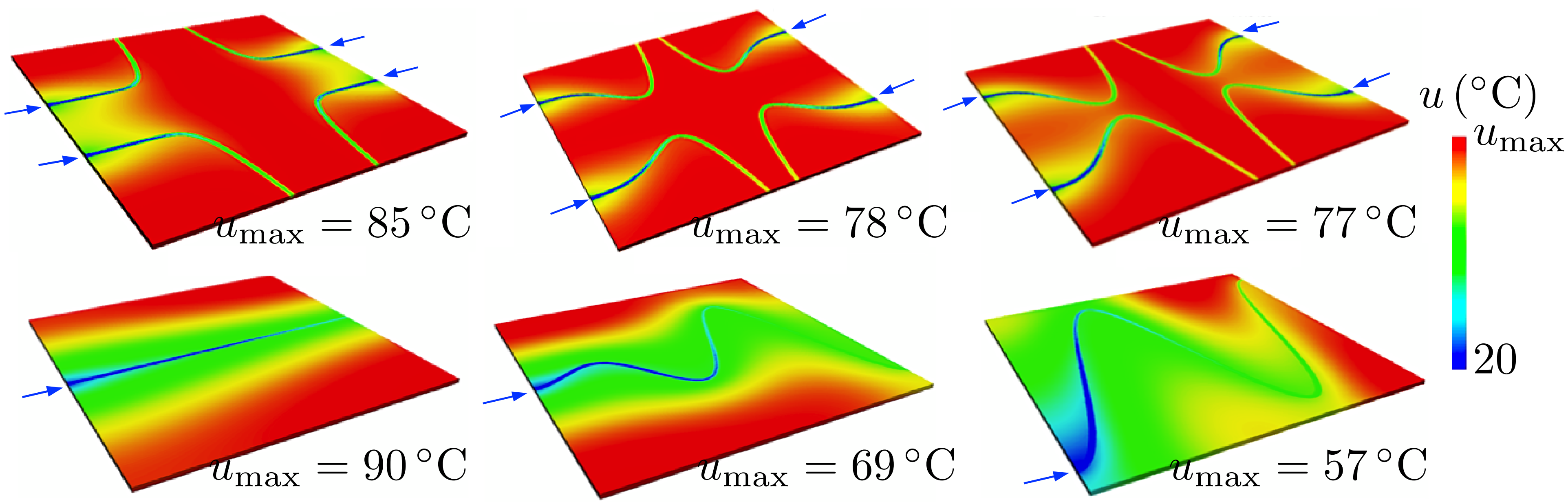
- Damage in complex microstructure:
  - 2.5 million degrees of freedom;
  - Condition number 5.6×10^{11} to 6.1×10^{4} (same as standard FEM).



- Aragón *et al.*, On the stability and interpolating properties of the Hierarchical Interface-enriched Finite Element Method. *Comput Methods Appl Mech Eng*, 362 (2020)

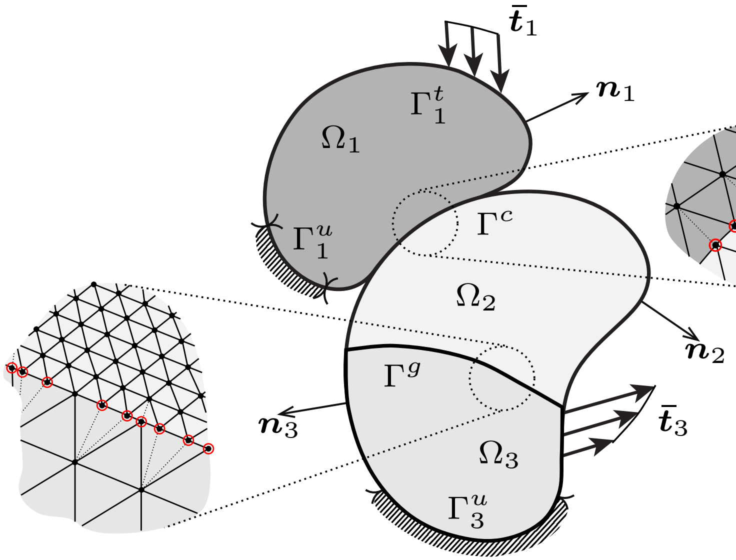
The formulation was used to simulate active cooling in an aluminum plate

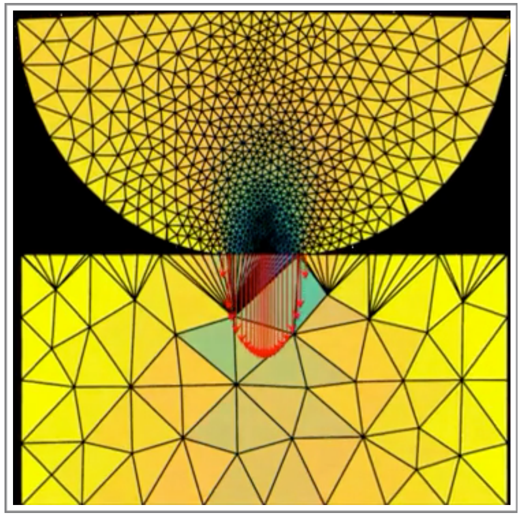
- Plates of 50x50x2 (mm^3):
  - Maximum (theoretical) temperature of 5,700 ℃;
  - Cooling a flow rate of 10 ml/min (top) or 40 ml/min (bottom):

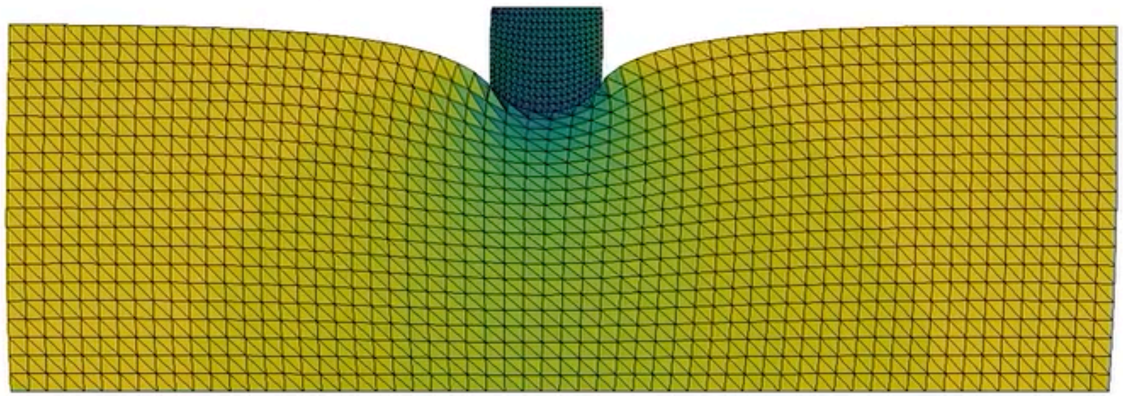


- Aragón *et al.*, On the stability and interpolating properties of the Hierarchical Interface-enriched Finite Element Method. *Comput Methods Appl Mech Eng*, 362 (2020)

The enriched formulation also works for contact and non-conforming mesh coupling





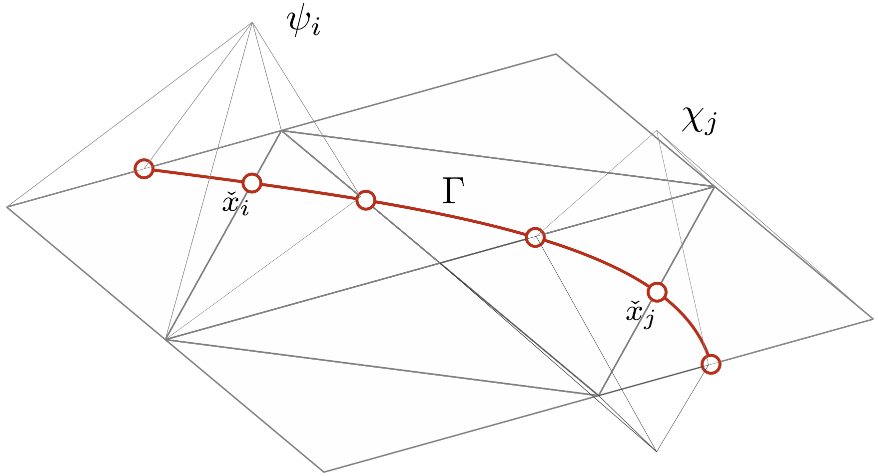


- Liu *et al.*, An interface-enriched generalized finite element formulation for locking-free coupling of non-conforming discretizations and contact, *Comput Methods Appl Mech Eng*, In Preparation.

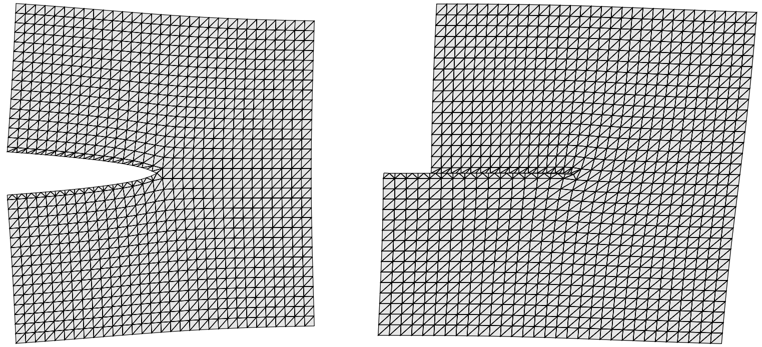
The interface-enriched formulation was also generalized to treat strong discontinuities

- Discontinuity-Enriched Finite Element Method (DE-FEM):

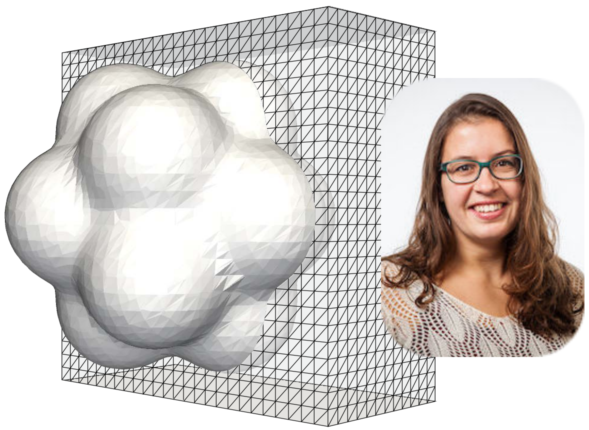
$$\boldsymbol{u}^h(\boldsymbol{x}) = \underbrace{\sum_{i \in \iota_h}^{n} N_i(\boldsymbol{x}) \boldsymbol{U}_i}_{\text{standard FEM}} + \underbrace{\overbrace{\sum_{i \in \iota_w} \psi_i(\boldsymbol{x}) \boldsymbol{\alpha}_i}^{\text{weak}} + \overbrace{\sum_{i \in \iota_s} \chi_i(\boldsymbol{x}) \boldsymbol{\beta}_i}^{\text{strong}}}_{\text{enriched or generalized}}$$



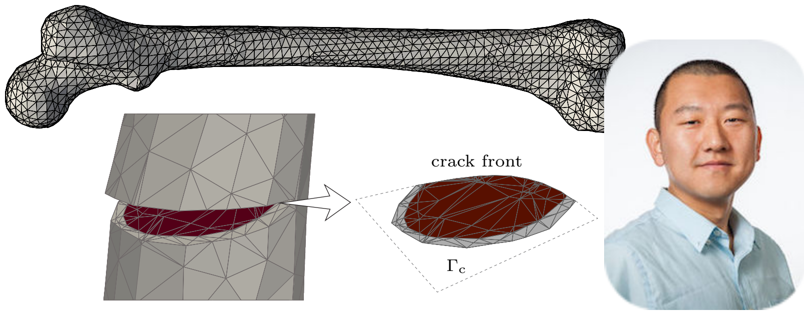
- Aragón and Simone, The Discontinuity-Enriched Finite Element Method. *Int J Numer Meth Eng*, 112 (2017)



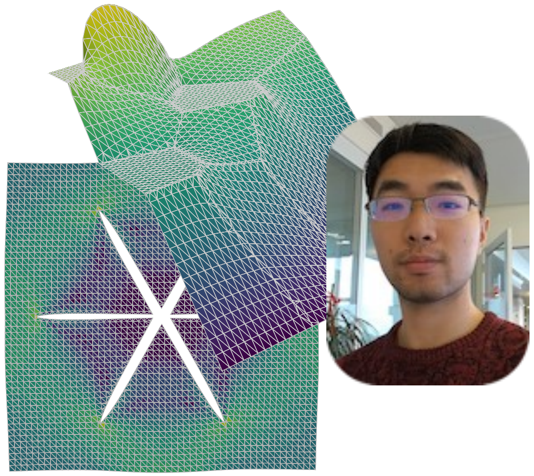
Discontinuity-Enriched FEM
Aragón and Simone, *IJNME* (2017)



Immersed boundaries
Sanne van den Boom

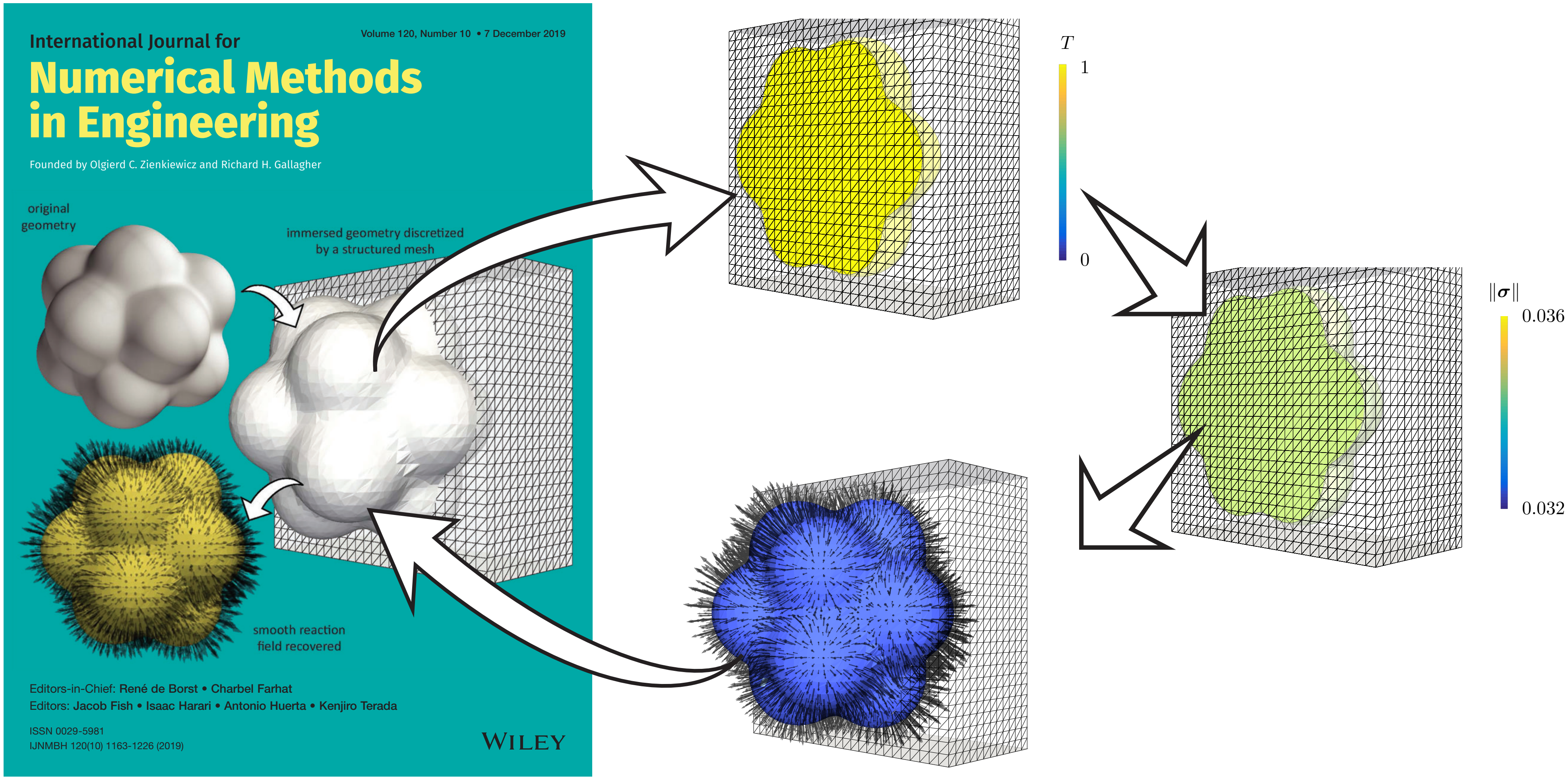


fracture mechanics
Jian Zhang



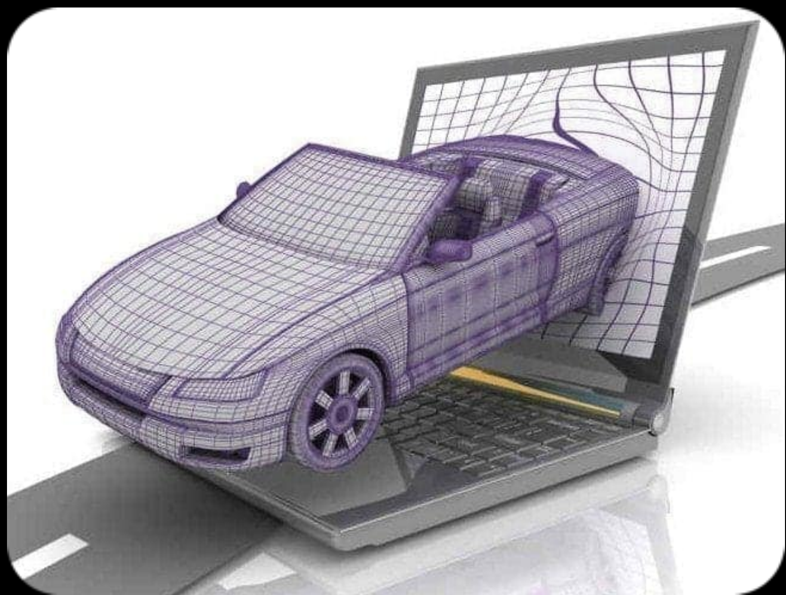
complex microstructures
Dongyu Liu

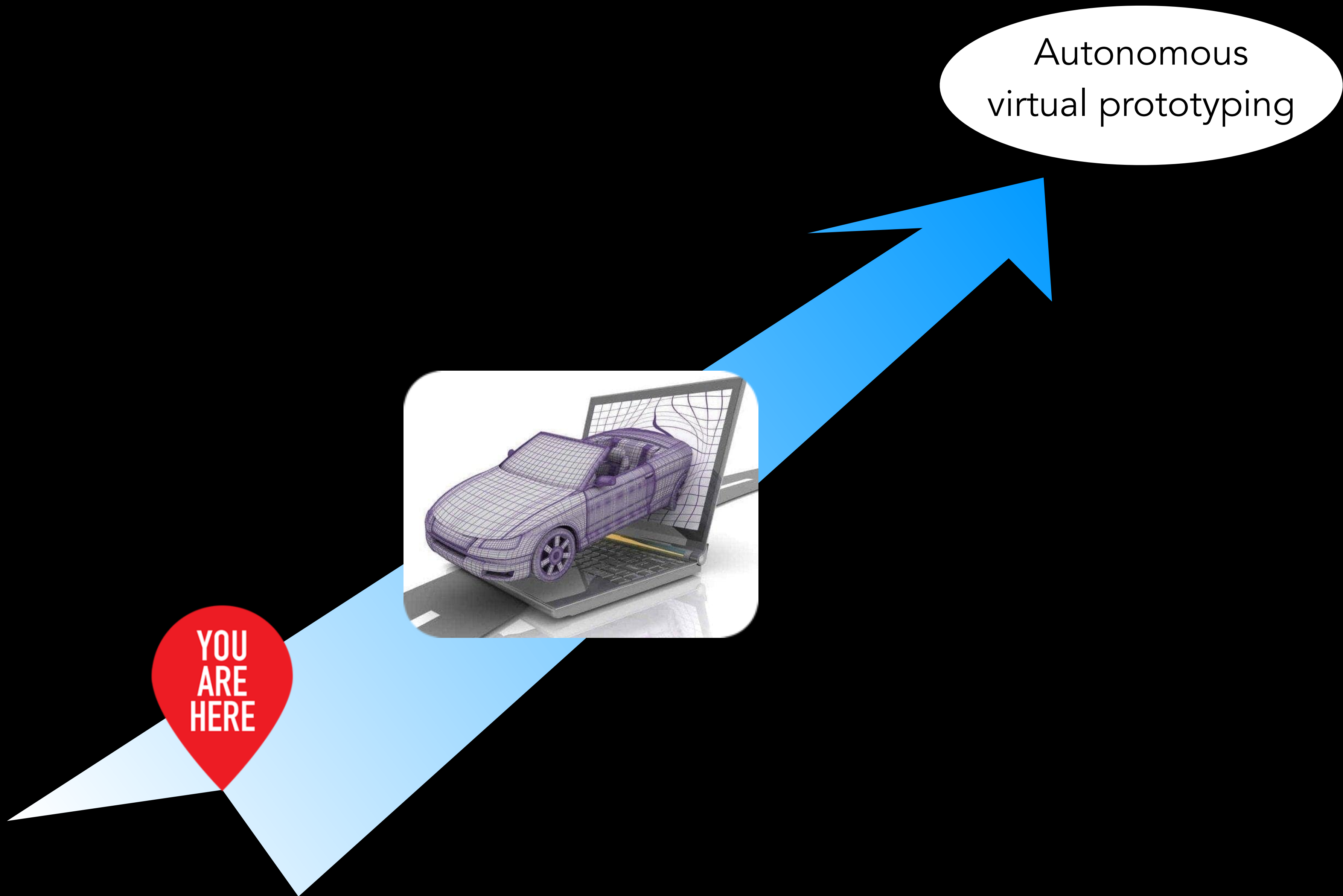
Enriched FEM was developed for immersed boundary (fictitious domain) problems



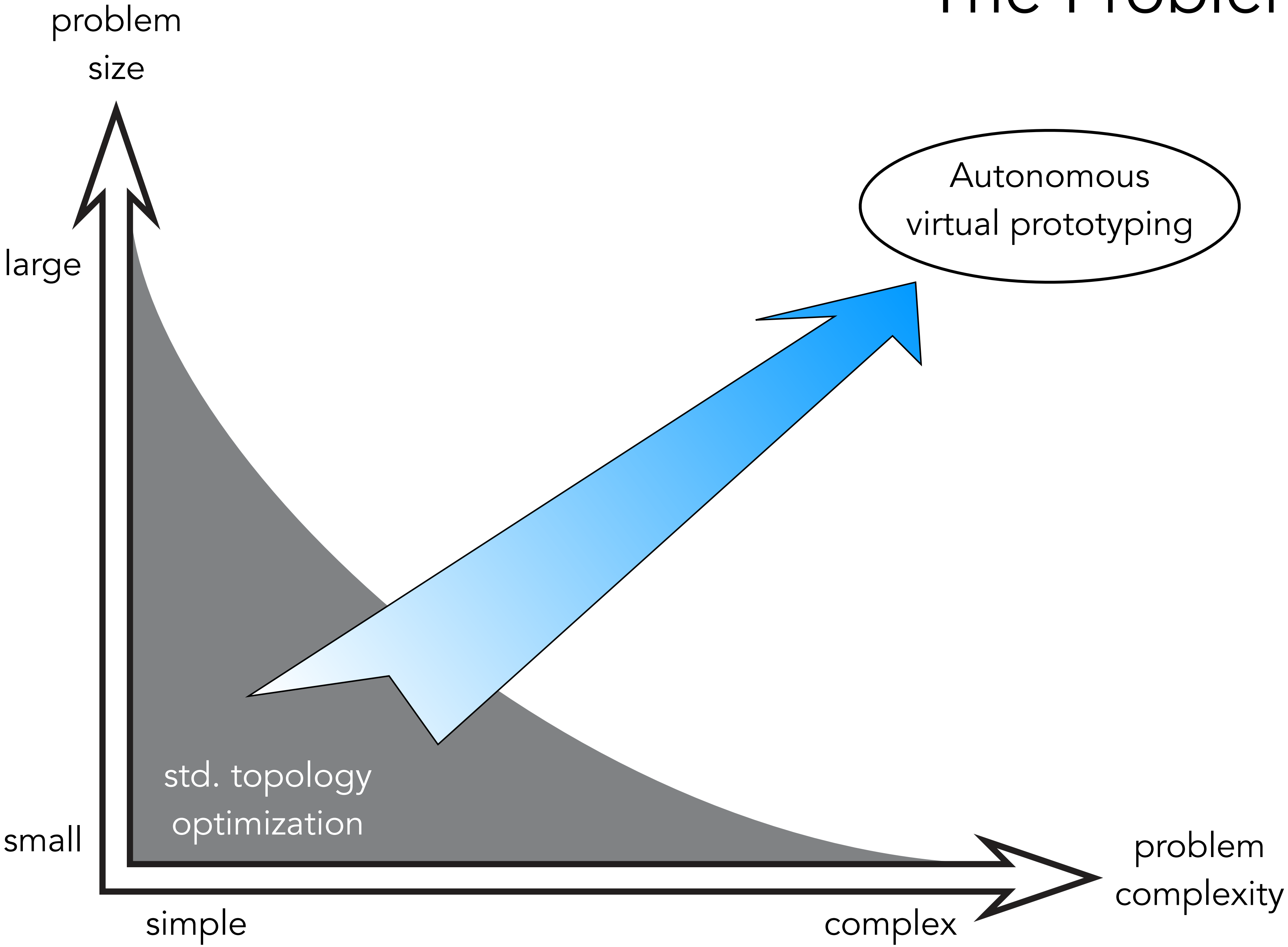
- van den Boom *et al.*, and Simone, A stable interface-enriched formulation for immersed domains with strong enforcement of essential boundary conditions. *Int J Numer Meth Eng*, 120 (2019)

Can we use enriched FEM for more than just analysis?

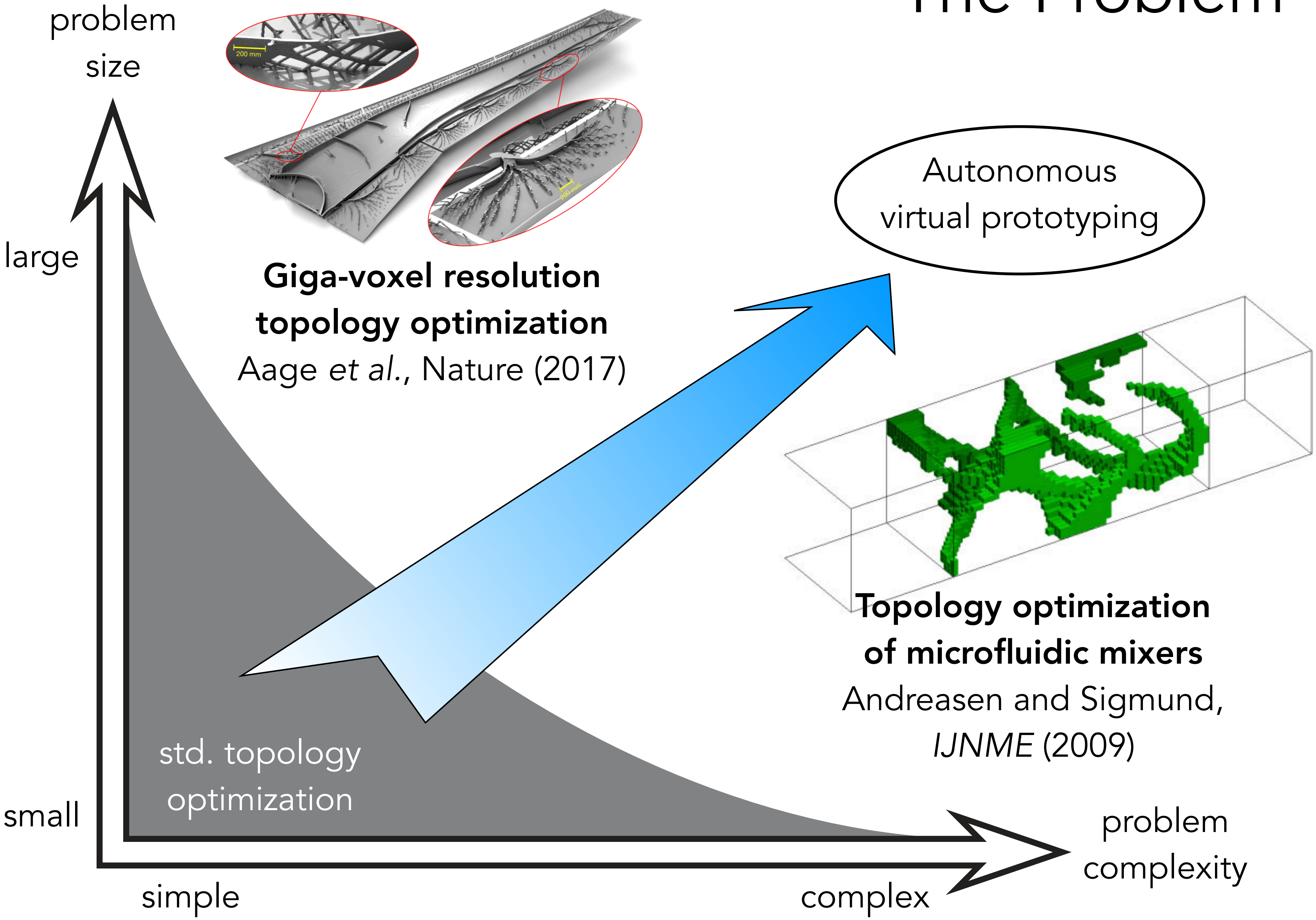



Autonomous
virtual prototyping
YOU
ARE
HERE

The Problem


problem
size
large
small
Autonomous
virtual prototyping
std. topology
optimization
simple
complex
problem
complexity

The Problem



problem
size
large
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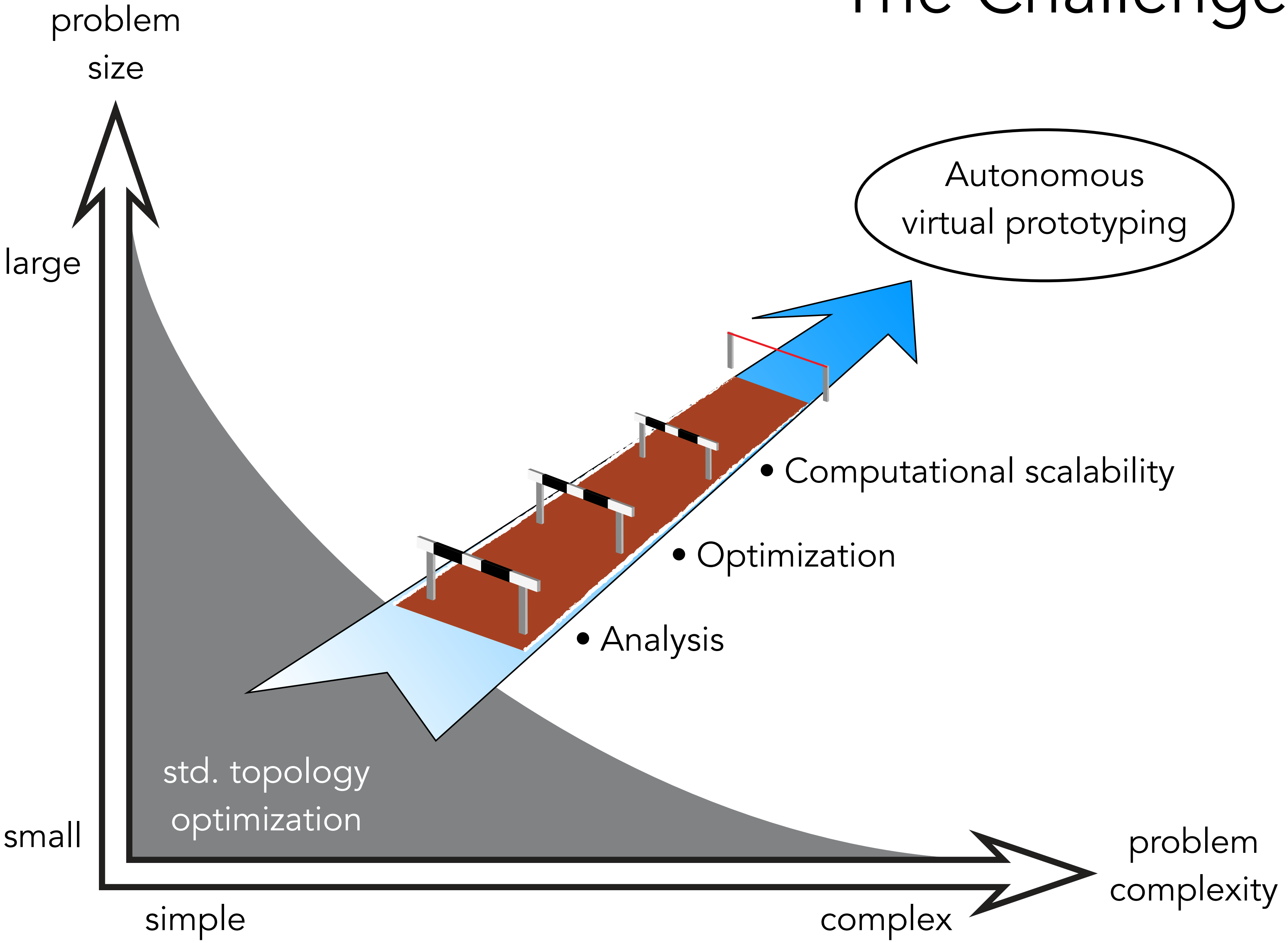
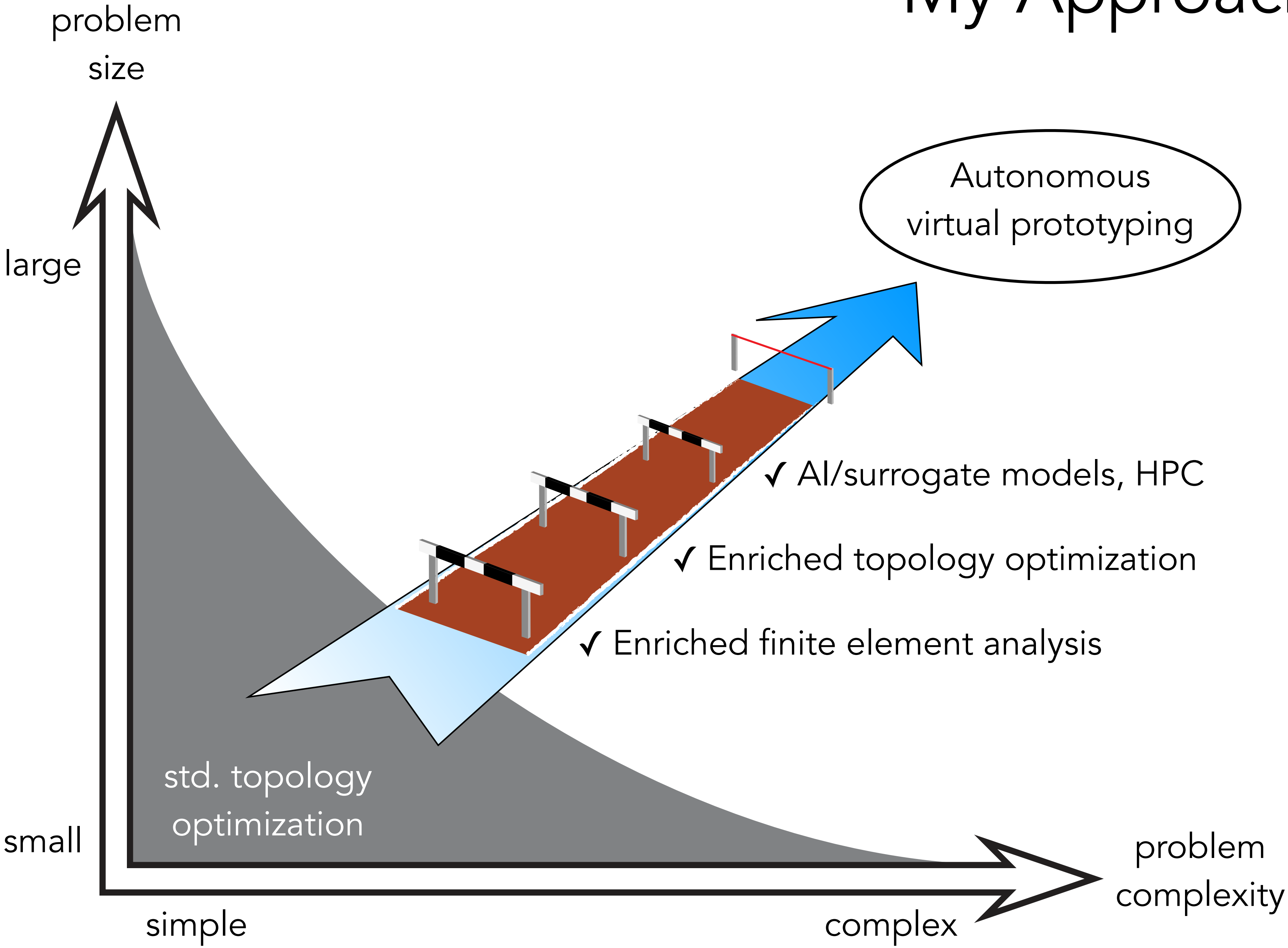
simple
complex
problem
complexity
std. topology
optimization
200 mm
Giga-voxel resolution
topology optimization
Aage *et al.*, Nature (2017)
Autonomous
virtual prototyping
Topology optimization
of microfluidic mixers
Andreasen and Sigmund,
IJNME (2009)

The Challenges



My Approach



problem
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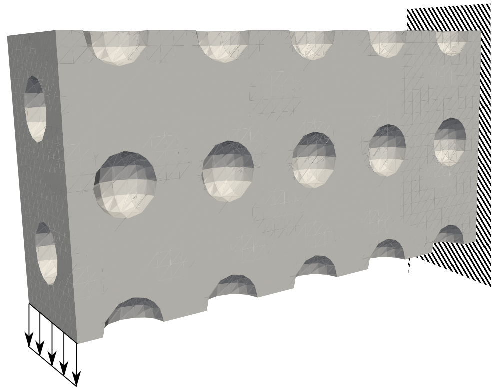
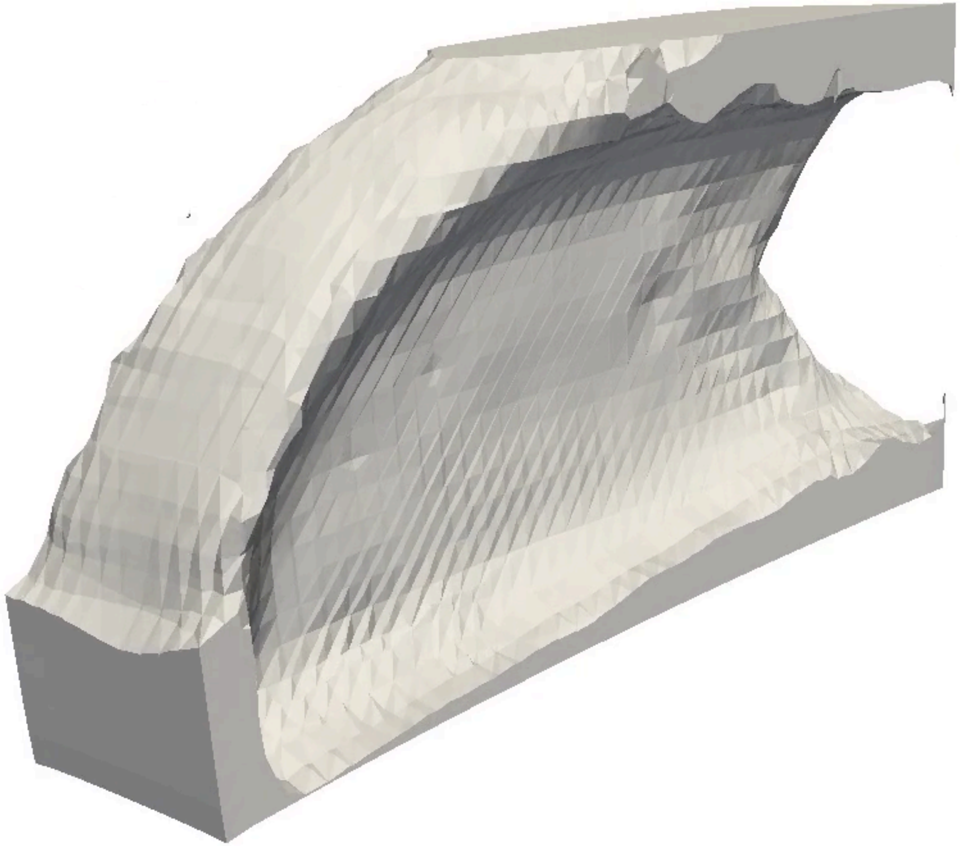
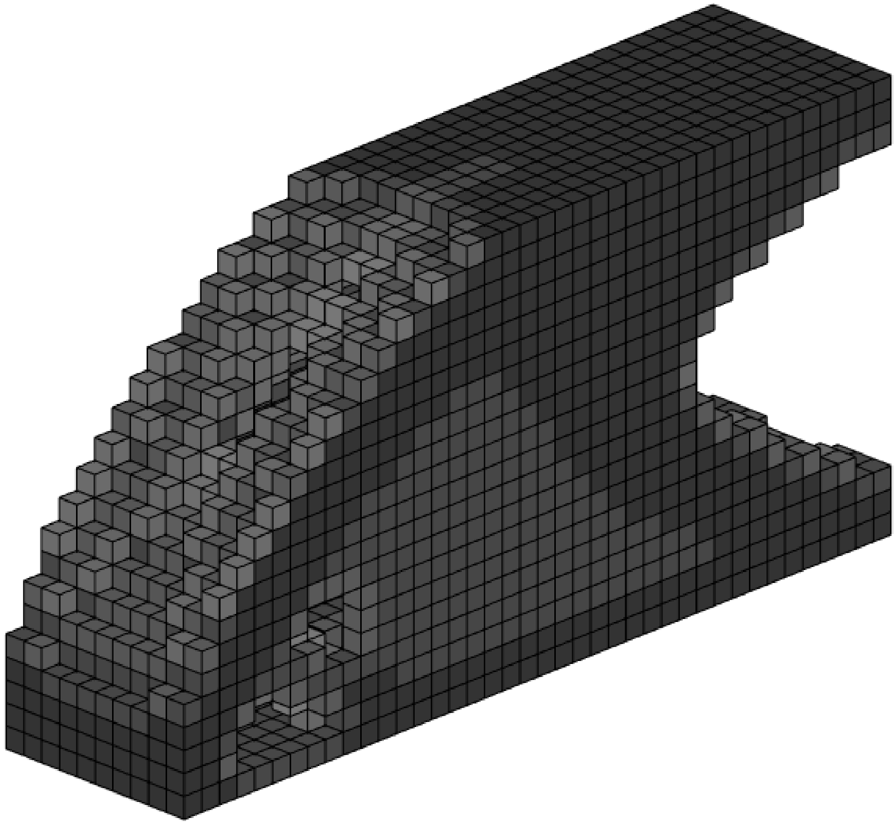
std. topology
optimization
Autonomous
virtual prototyping
✓ AI/surrogate models, HPC
✓ Enriched topology optimization
✓ Enriched finite element analysis

Enriched topology optimization
Sanne van den Boom

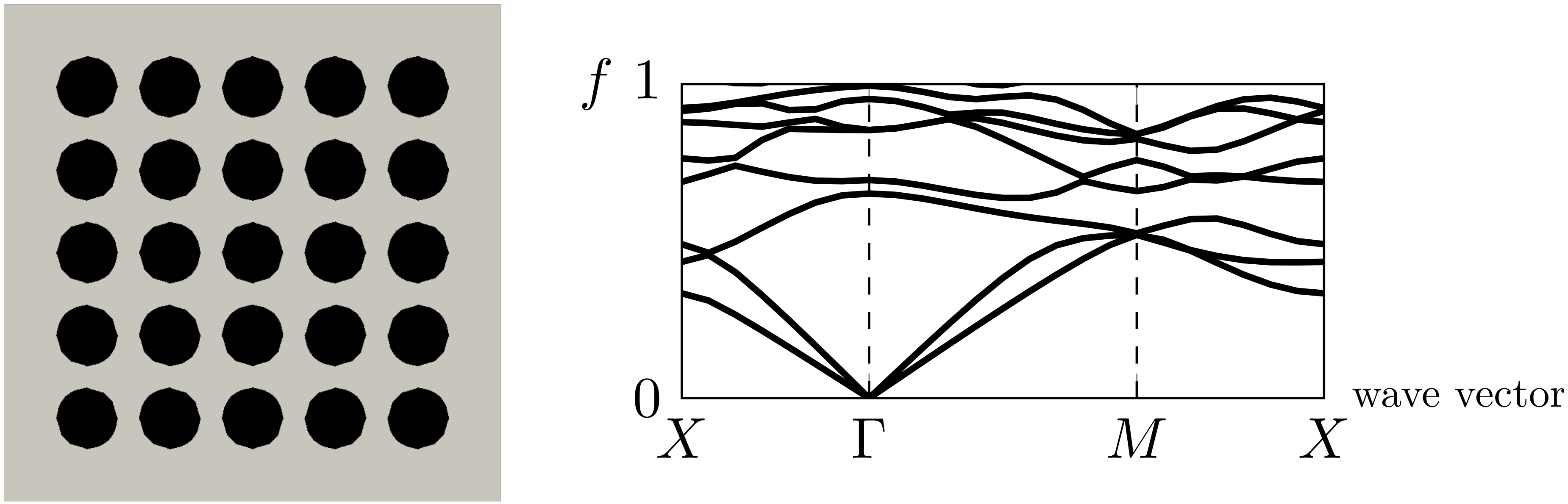






- van den Boom *et al.*, An Interface-enriched Generalized Finite Element Method for Levelset-based Topology Optimization. *Struct Multidiscipl Optim*, In Press.

We use enriched topology optimization to analyze and design phononic crystals



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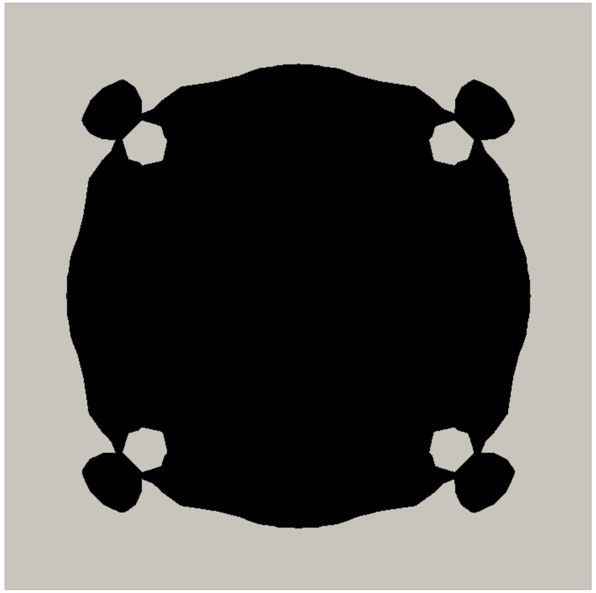
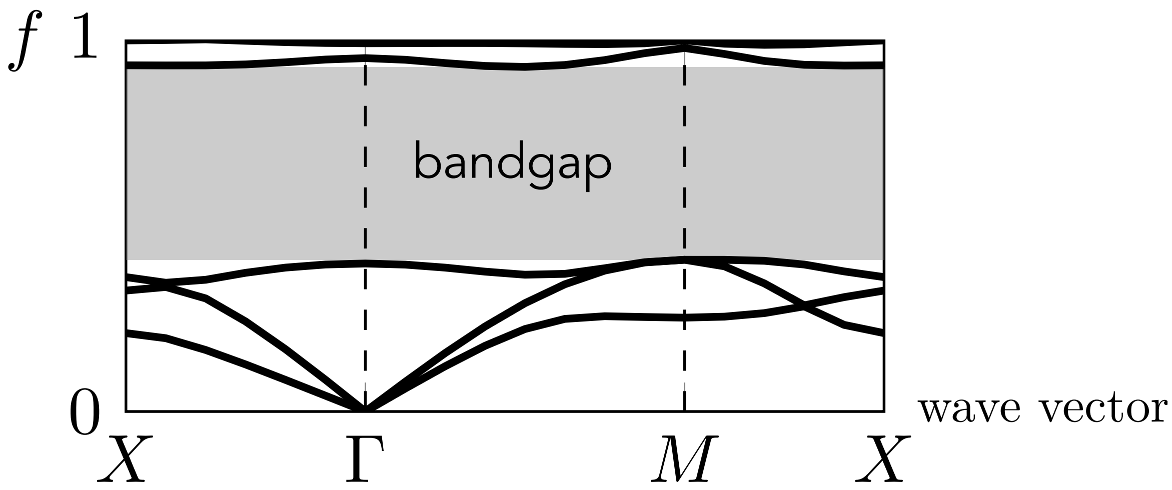




We have used enriched topology optimization for fracture anisotropy in 3D-printed chocolate

- Minimize/maximize energy release rate:

$$J = \omega \frac{1}{N} \sum_{i=1}^{N} G_{1i} - (1-\omega) \frac{1}{N} \sum_{i=1}^{N} G_{2i}$$

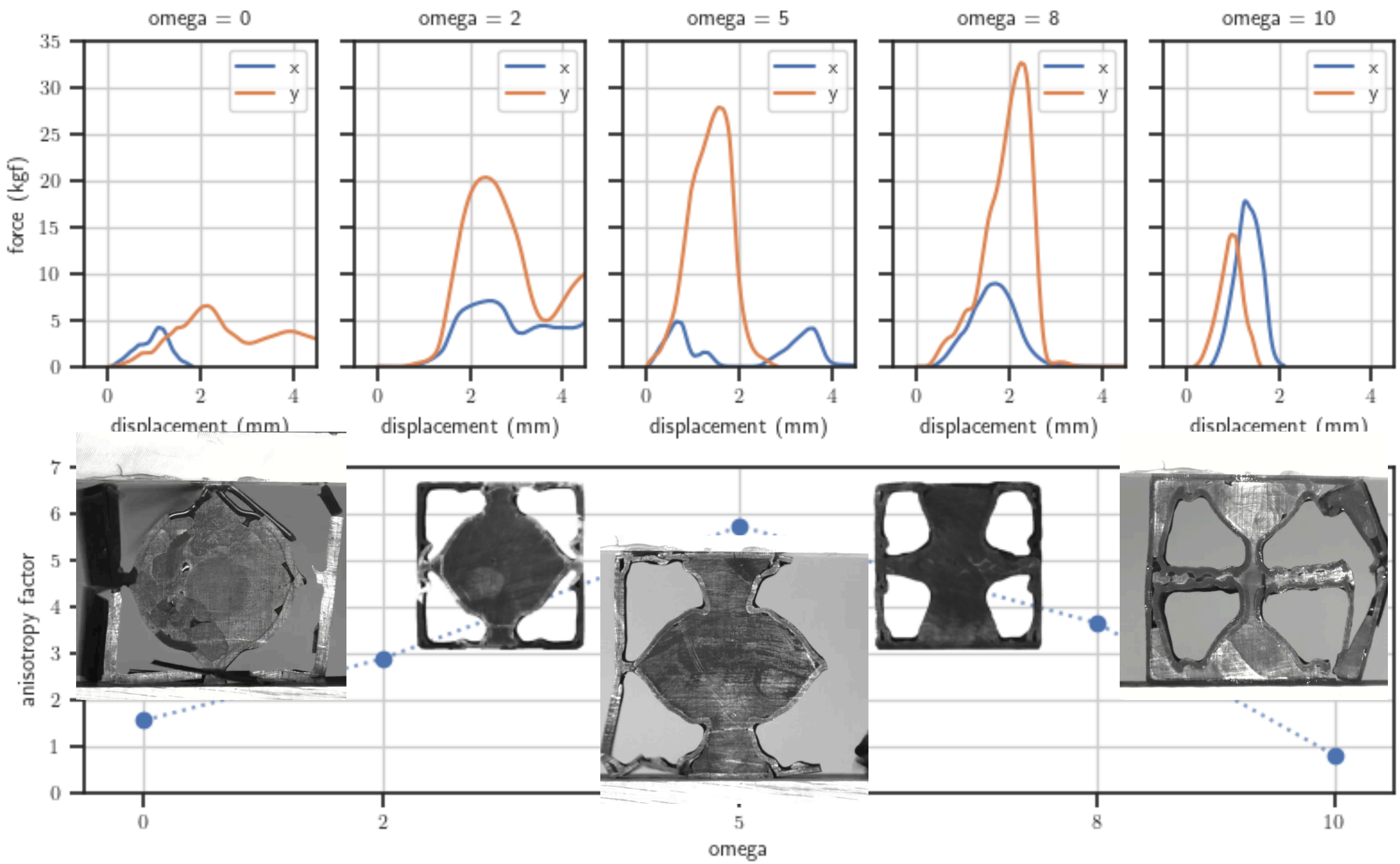
$\omega = 0$

$\omega = 0.5$

$\omega = 1$

Dr. Corentin Coulais
University of Amsterdam

We have used this technique to optimize for fracture anisotropy in chocolate



Conclusions

Enriched FEM…

- decouples discontinuities (interfaces, cracks) from discretization;
- can analyze immersed boundaries (fictitious domain) problems;
- can analyze numerical interfaces (coupling of non-conforming meshes) and contact with proper transfer of tractions;
- is stable and yields the same accuracy as standard FEM with fitted/ matching meshes;
- can effectively be used for topology optimization in combination with a parametrized level set;
- for topology optimization yields smooth *black-and-white* designs that are free from *staircased/pixelized* boundaries;

Thank you…

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