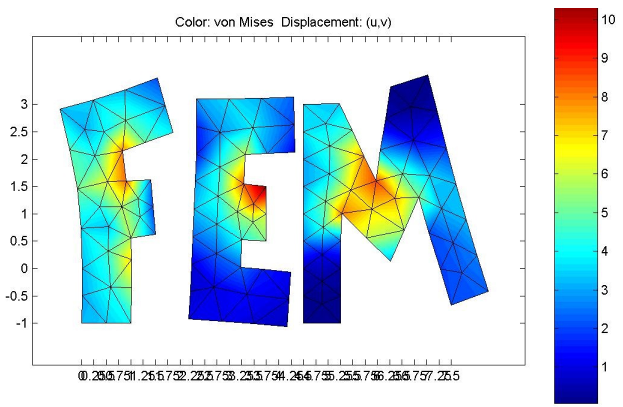
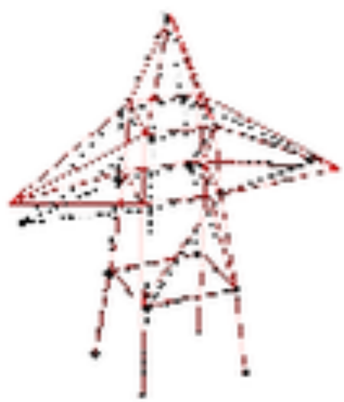
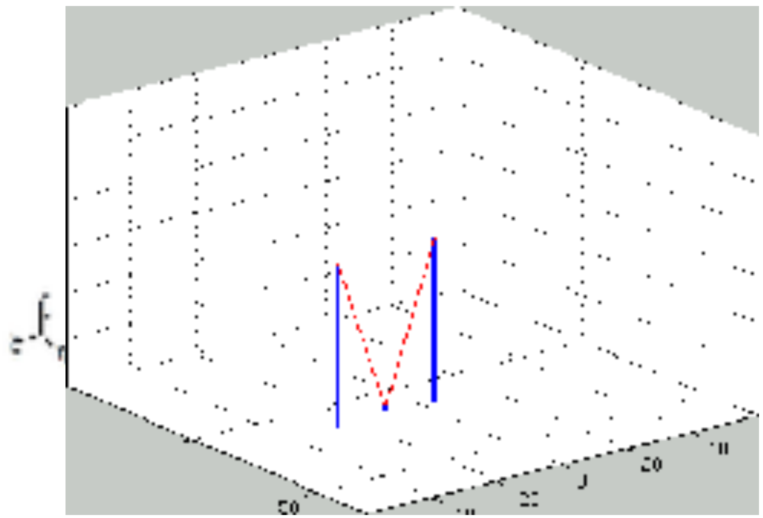
INTEGRATION OF SIMULATION BASED MATHEMATICS IN MECHANICAL ENGINEERING







```
[TOUT,YOUT] = ODE23(ODEFUN,TSPAN,Y0)
```

Mikael Enelund,
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OUTLINE

- Integration of simulation based mathematics education
- Background
- Reformed Education
- Program learning outcomes and program design
- Case studies
- Evaluation and results

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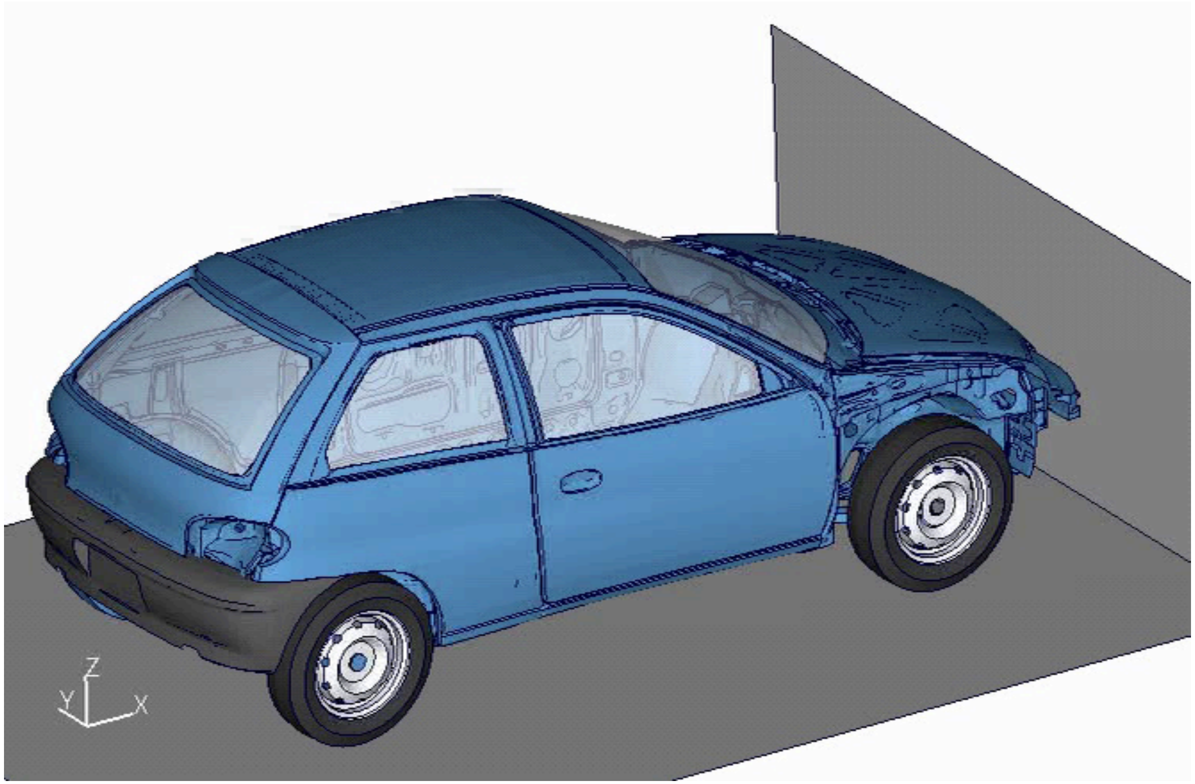
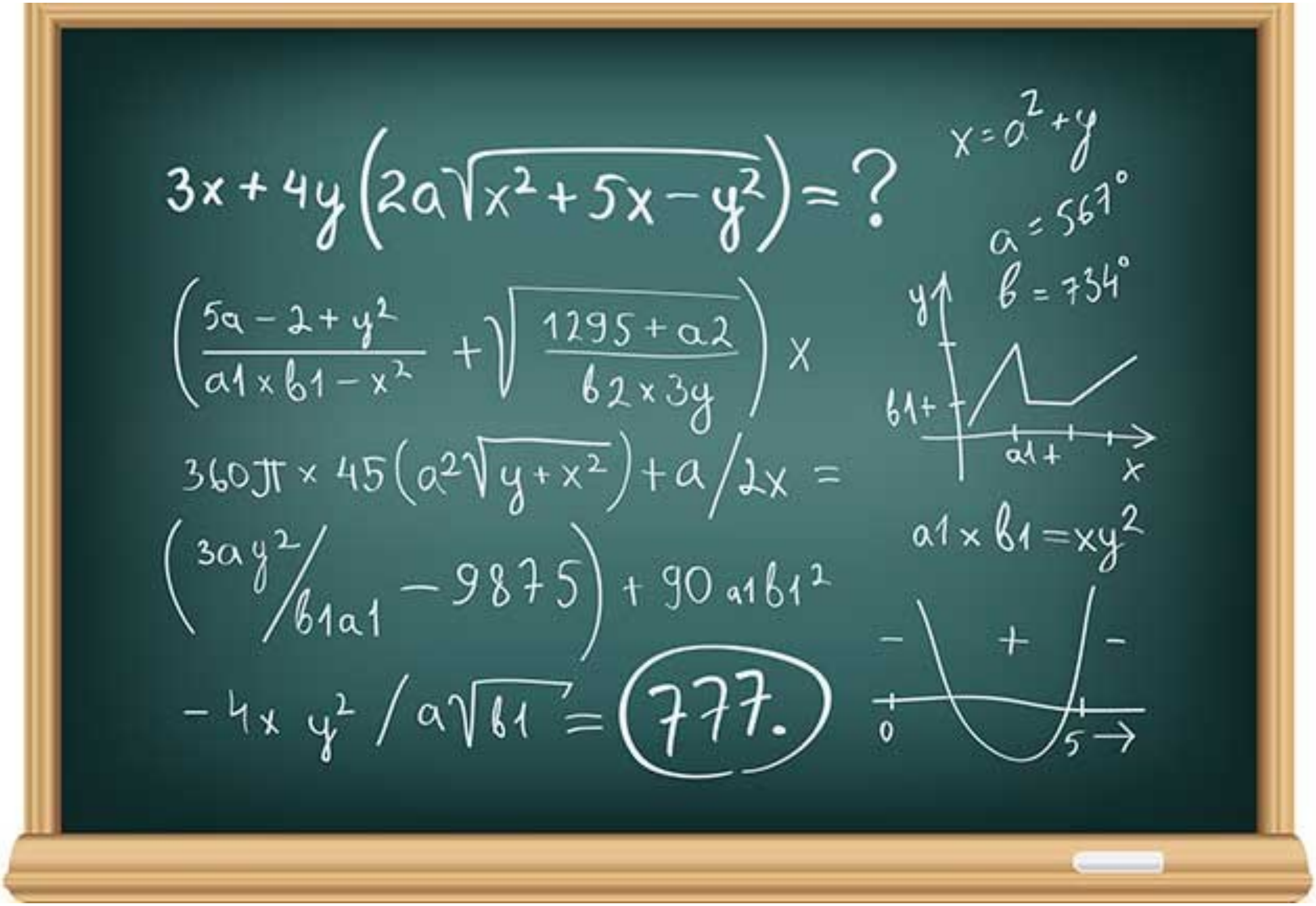
ME ENGINEERS USE A LOT OF ADVANCED MATH



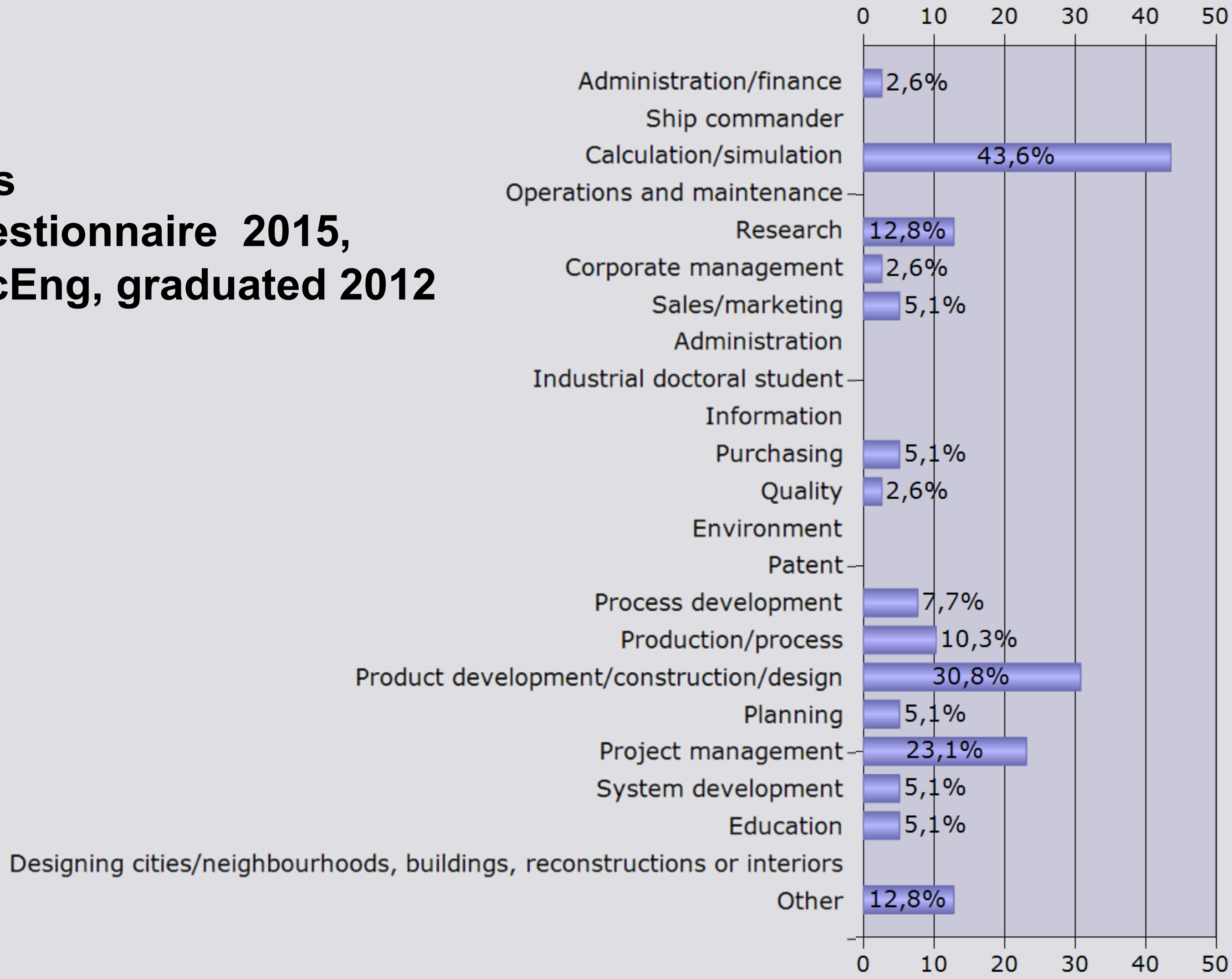
- Simulation driven design
- CAE
- Optimization
- Control
- Industry 4.0
- Internet of things
- Computerization and digitalization
- Programming
- …..

That kind of mathematics is historically not taught…

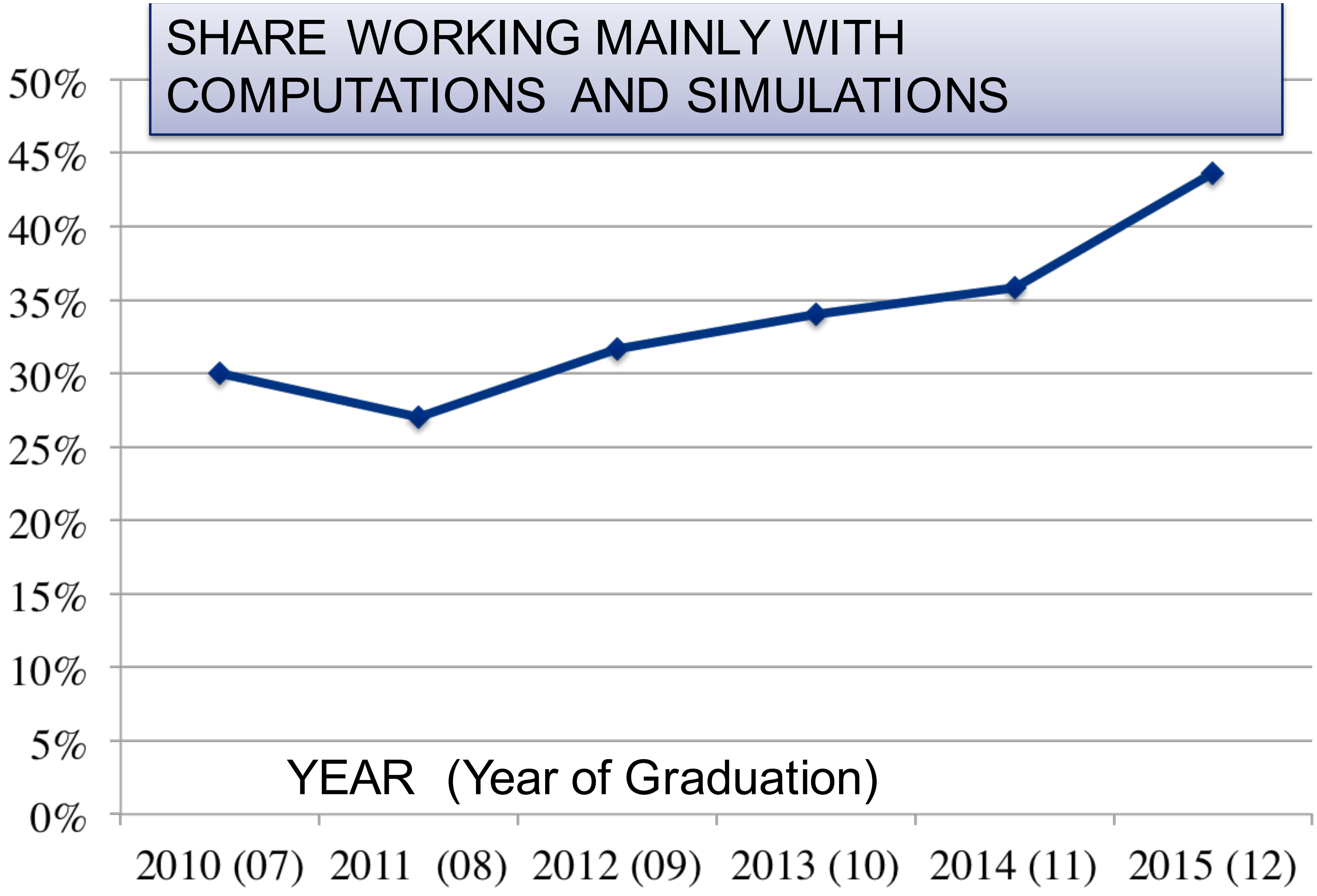


What do mechanical engineers three years after graduation?

Main Duties
Alumni questionnaire 2015,
MSc in MecEng, graduated 2012



What do mechanical engineers three years after graduation?



BACKGROUND – REFORMED MATH

- Rapid development of computers and the internet
  - Solving most problems faced in modern engineering includes high precision digital models and simulations.
  - Preparing students for a modern approach based on modelling, simulation and analysis
- CDIO raises the need for a reform of math education
  - Need for a toolbox to handle real (complex) problems
- Young persons learn much differently than they used to a few decades ago. “the Nintendo Syndrome”
  - Do not read manuals but go and try. If “killed” try something else until you get to the next level. If no success go to the internet to get a hint.
  - This comes through in studying: it is becoming increasingly hard to make students read books. At the same time providing them with ways to try before reading proves educationally rewarding.
- Need for reformed math education, not less math

BACKGROUND, CONT’D

- Mathematics in a CDIO-program
  - Disciplinary knowledge (Standards 1)
  - Full view of problem solving (Standards 2)
  - Modelling, decision making and design (Standards 2-4 and 6)
  - Active learning techniques: Interactive and virtual learning environments (Standards 8)
  - The need for virtual prototyping lab that is: Need for engineering tools for calculation and simulation
  - CDIO-framework beneficial for designing an integrated program (cf integration of general skills)

REFORMED SIMULATION BASED MATHEMATICAL EDUCATION

- Launched 2006/2007 and continuously improved,
- New math courses including a basic course in Matlab programming,
- Focus transferred from solving oversimplified special problems with known solutions to more open general problems,
- Interactive/virtual learning learning environments
- Teaching and learning in computer lab,
- Textbook book in Computational math. Programming, numerics and simulations integrated,
- Integration of mathematics in other fundamental engineering courses,

CORNERSTONES

- To highlight and clarify modelling, computations, analyses and simulations,
- Full integration of computational aspects (including programming) and symbolic aspects of mathematics,
- Construction of algorithms and writing own programs (programming skills and understanding of mathematics and algorithm construction)
- General equations instead of the simplified special equations whose solutions can be written in elementary functions
- The finite element taught in first year math course Calculus in Several Variables and used in Solid Mechanics course
- Computer-oriented exercises, assignments and team projects that are used simultaneously in the mathematics courses and in courses of mechanics and solid mechanics

PROGRAM LEARNING OUTCOMES: MATH

The Master of Science in Mechanical Engineering graduate shall:

1 Be able to put into practice (apply) mathematics with focus on being able to

1.1 solve linear and nonlinear systems of algebraic equations by numerical methods,

1.2 solve ordinary differential equations of the following types; separable, inhomogeneous with constant coefficients and Euler's,

1.3 solve by numerical methods linear and nonlinear ordinary differential equations inclusive reformulating to a first order system,

1.4 solve the eigenvalue problem for continuous and discretized systems

1.5 use the Finite element method to solve partial differential equations,

1.6 explain the fundamentals of probability theory and statistics and being able to plan experiments with respect to statistical variations

1.7 program solutions, including graphic presentations of engineering problems in Matlab and/or Python.

PROGRAM LEARNING OUTCOMES – MATH, CONT’D

The Master of Science in Mechanical Engineering graduate shall:

2 Be able to formulate theoretical models and set up equations to describe the models. Solve equations in order to simulate reality and assess the reasonableness of the choice of model and the solution's level of accuracy.

3 Be able to analyze, solve and simulate advanced mechanical engineering problems within the selected specialization area/master's program by using modern, computer-based tools and from these, selecting the most appropriate ones

The reformed courses are developed to meet these goals.

Goals MATH **Courses (mandatory)**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**2**	**3**
Introductory course in Mathematics	W, CE						CE		
Programming in Matlab							W, CE		
Calculus in a Single Variable	W	W, CE	W, CE				CE		
Computer Aided Design									W, P
Linear Algebra	W, CE			W, CE			CE		
Statics and Solid Mechanics	W CE	W					CE	CE	
Calculus in Several Variables	W, CE	W, CE		W, CE	W, CE		CE		
Solid Mechanics	W, CE	W, CE			W, CE		CE	CE	CE
Mechanics – Dynamics	W	W, CE	W, CE				CE	CE	CE
Engineering Materials									
Machine Elements	W	W					P	P	
Material and Manufact. Techn.					P				
Thermodynamics and Energy Technology							P	P	
Sustainable Product Develop.									
Integrated Design and Manufact.								P	P
Mechatronics							CE	CE	
Fluid Mechanics	W, CE	CE	CE				CE	CE	L
Automatic Control	W, CE	W, CE	W, CE				CE	CE	CE
Mathematical Statistics						W, P	P	P	P

Part of program design matrix – math assessment

W = Written exam/quiz
CE = Computer exercise
P = Project
L=Lab

Systematic approach to evaluate the program learning outcomes

Goals MATH Courses (mandatory)	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2	3
Introduction to Mechanical Eng.									
Introductory course in Mathematics	T	I						T	
Programming in Matlab	T							T	I
Calculus in a Single Variable	T	T	T						
Computer Aided Design								T	T
Linear Algebra	T		T	T				T	
Statics and Solid Mechanics	U	U	U					T	
Calculus in Several Variables	U	T	T	T	T			T	
Solid Mechanics	U	U	U		T			T	T
Mechanics – Dynamics	U	U	U	U				U	
Engineering Materials	U	U		U				U	T
Machine Elements	U	U	U	U	U			U	U
Material and Manufact. Techn.	U	U	U	U	U			T	U
Thermodynamics and Energy Technology	U	U					T		
Sustainable Product Develop.								U	
Integrated Design and Manufact.	U	U							T
Industrial Production and Org.									
Engineering Economics									
Mechatronics	U	U	U					T	
Fluid Mechanics	U	U	U	U	U		U		T
Automatic Control	U	U	U	U					
Mathematical Statistics	U	U	U			T			T
Bacheleor Diploma Project									U

Part of program design matrix – teaching math

I = Introduce
T = Teach
U = Utilize

Systematic approach to design an integrated curriculum

ME PROGRAMME – INTERGRATED CURRICULUM

YEAR 1

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Programming in Matlab (4.0)	Computer aided engineering (4,0)	**Linear algebra** (7,5)	**Mathematical analysis in several variables** (7,5)
Introductory mathematics (7,5)	**Mathematical analysis in a single variable** (7.5)	Statics & strength of materials (7.5)	Solid Mechanics (7,5)
Introduction to Mechanical Engineering (7,0)			

Joint exercises/assignments/projects

Matlab programming, numerical solutions and simulations

Simulation using industrial software (CATIA, ANSYS, ADAMS, FLUENT…)

YEAR 2

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Mechanics - Dynamics (7,5p)	Machine element (7,5)	Thermodynamics and energy technology (7,5)	Industrial production and organisation (6)
Material technology (7,5)	Material and manufacturing technology (7,5)	Integrated design and manufacturing (7,5)	
		Sustainable product development (4,5)	Engineering economics (4,5)

- Joint exercises/assignments
- Matlab programming, numerical solutions and simulations
- Simulation using industrial softwares (CATIA, ANSYS, ADAM, FLUENT…)

YEAR 3

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Mechatronics (7,5)	Automatic control (7,5)	Bacheleor diploma project (15)	
Fluid mechanics (7,5)	Elective 1 (7,5)	Elective 2 (7,5)	Mathematical statistics (7,5)

Elective 1

- Energy conversion
- Finite element method
- Machine design
- Simulation of production

Elective 2

- Logistics
- Sound and vibration
- Material and process selection
- Objectoriented programming
- Transforms and differential equations
- Heat transfer

Matlab programming, numerical solutions and simulations

Simulation using industrial softwares (CATIA, ANSYS, ADAMS, FLUENT…)

FIRST YEAR MATH COURSES

Quarter 1: Introductory Mathematics

Function, continuity, derivative in one variable. Series.

Computer exercises:

1. Function gallery
2. Bisection algorithm
3. Fixed point iteration
4. Numerical derivative
5. Newton's method

Quarter 2: Mathematical Analysis in One Variable

Integral, ordinary differential equation. Geometry and vector algebra.

Computer exercises:

1. ODE1: primitive function (integral)
2. ODE2: Euler's method for systems of ODE
3. ODE3: implicite methods
4. ODE4: boundary value problems (shooting methods with Euler Solver)

FIRST YEAR MATH COURSES, CONT'D

Quarter 3: Linear Algebra (in Parallel with Statics and Strength of Materials)

Gauss elimination, determinant, inverse matrix.
Orhtogonality, eigenvalue problem. Least squares method.
Computer exercises:

1. Matrix algebra
2. Geometry
3. Systems of linear equations, error analysis, condition number (Elastic Truss)
4. Least squares (calibration of Norton's law for creeping)

FIRST YEAR MATH COURSES, CONT'D

Quarter 4: Mathematical Analysis in Several Variables (taught parallel with Solid Mechanics)

Partial derivative. Linearization, Jacobi matrix, Newton's metod. Taylor's formula. Optimization. Curves and surfaces. Double and triple integral. Curve integral, surface integral. Boundary value problems and the finite element method in one and several variables.

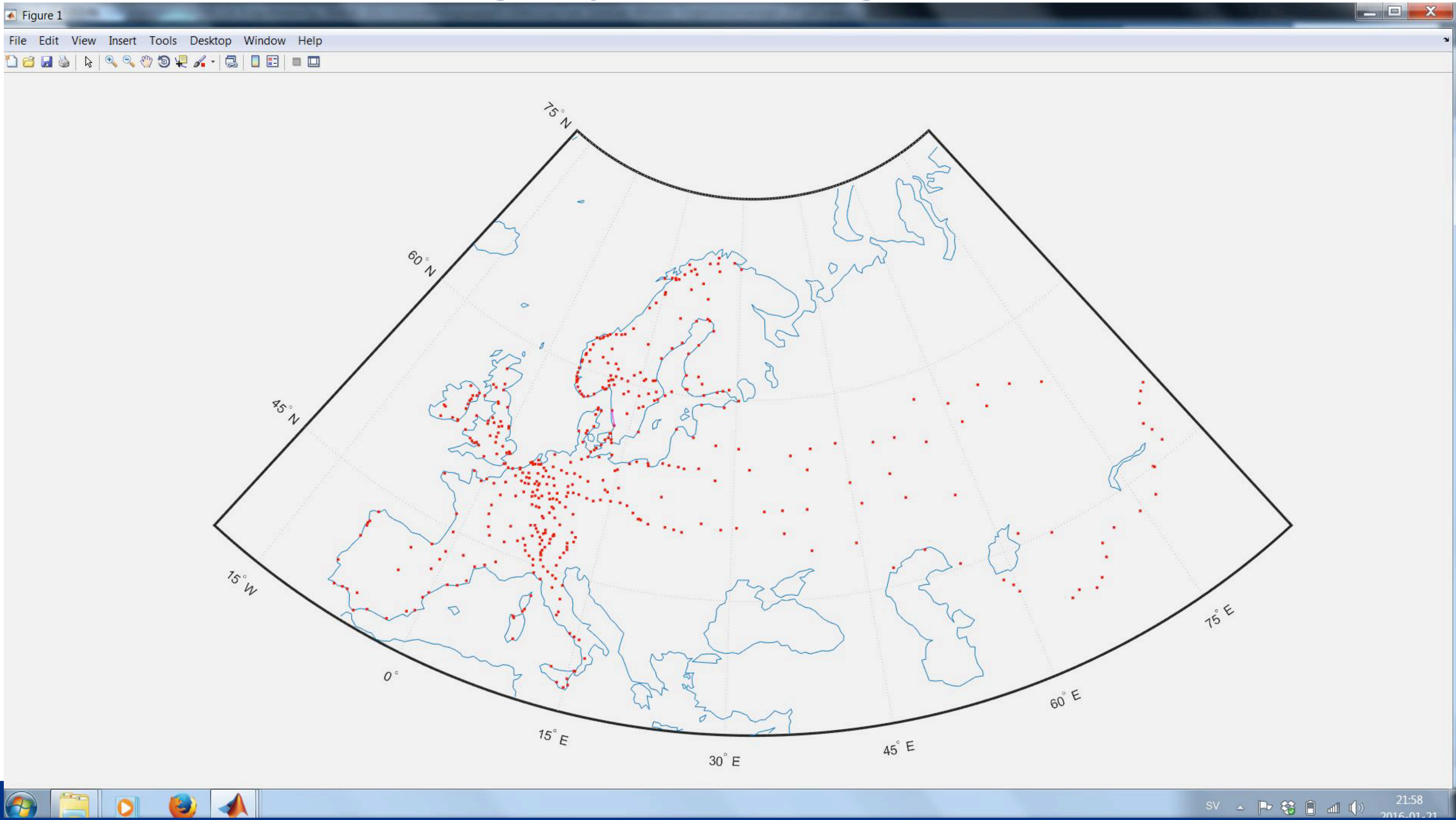
Computer exercises:

1. Visualization of multivariable functions
2. Jacobi matrix and Newton's method
3. Optimization
4. The finite element method in 1–D (own code)
5. The finite element method in 2–D (Matlab's PDE Toolbox, ANSYS)

PROGRAMMING IN MATLAB

- Aim: Develop own programs from problem descriptions.
- Easy to use and suitable as a first programming environment
- Used in all applied courses and in applied research
- Toolboxes and built-in functions
- Students may use Python (or any environment)
- Third year course in Object-oriented programing in Python

Example: Assignment "Least cost path using Dijkstra's algorithm"



Example: Introductory mathematics (taught in parallel with Programming) in Matlab

Compute square $\sqrt{2}$ by bisection algorithm $x_n = \frac{x_{n-1} + 2/x_{n-1}}{2}$

$$
\begin{aligned}
x_0 &= 1 \\
x_1 &= (x_0 + 2/x_0)/2 \approx 1.5 \\
x_2 &= (x_1 + 2/x_1)/2 \approx 1.416666666667 \\
x_3 &= (x_2 + 2/x_2)/2 \approx 1.414215686275 \\
x_4 &= (x_3 + 2/x_3)/2 \approx 1.414213562375 \\
x_5 &= (x_4 + 2/x_4)/2 \approx 1.414213562373 \\
x_6 &= (x_5 + 2/x_5)/2 \approx 1.414213562373 \\
x_7 &= (x_6 + 2/x_6)/2 \approx 1.414213562373 \\
x_8 &= (x_7 + 2/x_7)/2 \approx 1.414213562373 \\
x_9 &= (x_8 + 2/x_8)/2 \approx 1.414213562373 \\
x_{10} &= (x_9 + 2/x_9)/2 \approx 1.414213562373
\end{aligned}
$$

MATLAB

```
x   = 1;
tol = 1e-10;
dx = 2*tol;

while abs(dx) > tol
    dx = (x + 2/x) / 2 - x;
    x += dx;
end
```

PYTHON

```
x = 1.0
tol = 1e-10
dx = 2*tol

while abs(dx) > tol:
    dx = (x + 2/x) / 2 - x
    x += dx
```

JULIA

```julia
x = 1
tol = 1e-10
dx = 2*tol

while abs(dx) > tol
    dx = (x + 2/x) / 2 - x
    x += dx
end
```

C++

```cpp
int main()
{
  double x = 1.0;
  double tol = 1e-10;
  double dx = 2*tol;

  while (abs(dx) > tol)
  {
    dx = (x + 2/x) / 2 - x;
    x += dx;
  }

  return 0;
}
```

Example: Introductory mathematics

Write a program that solves the equation *f (x)* = 0 with arbitrary accuracy.

Solve $5x + 5 = 10, \quad x^2 = 2, \quad x^5 - x + 1 = 0, \quad x = \cos x$

Example: Mathematical analysis in several variables

Write an FE-program that solves the boundary value problem

$$-\frac{d^2u}{dr^2} - \frac{1}{r}\frac{du}{dr} + \frac{1}{r^2}u = \frac{1-\nu^2}{E}K_r \quad \text{for } r \in I = (a,b)$$

$$u(a) = 0, \; u'(b) = 0.$$

with $K_r = \rho\omega^2 r$.

Governing equation for a rotating elastic disc

Some Remarks

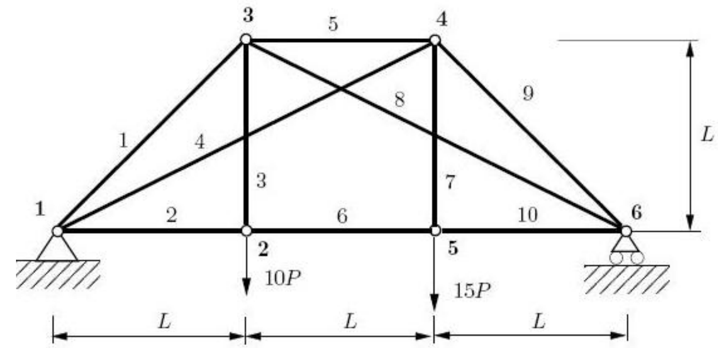
- Mathematics as a general tool for modeling
- Mathematics as a general tool for solving equations
- General methodology requires numerics + programming
- General methodology requires mathematical theory,
- General methodology same for $x^2 = 2$ as for systems of coupled nonlinear partial differential equations

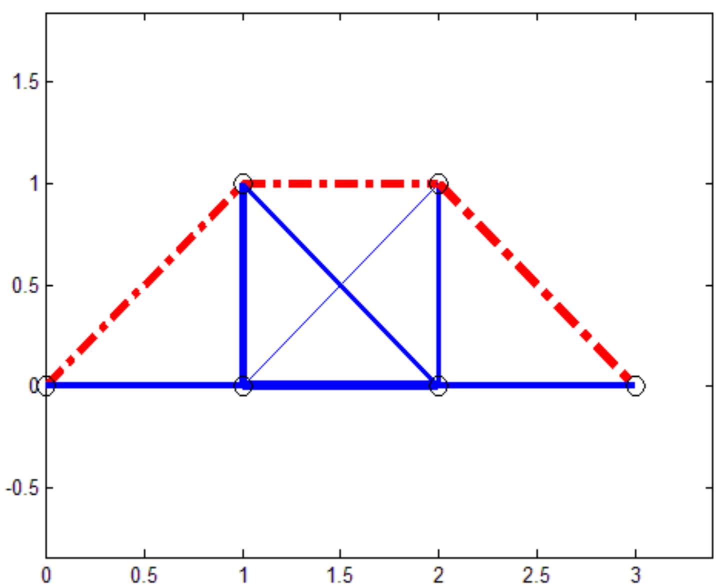
JOINT COMPUTER ASSIGNMENTS, 2 EXAMPLES

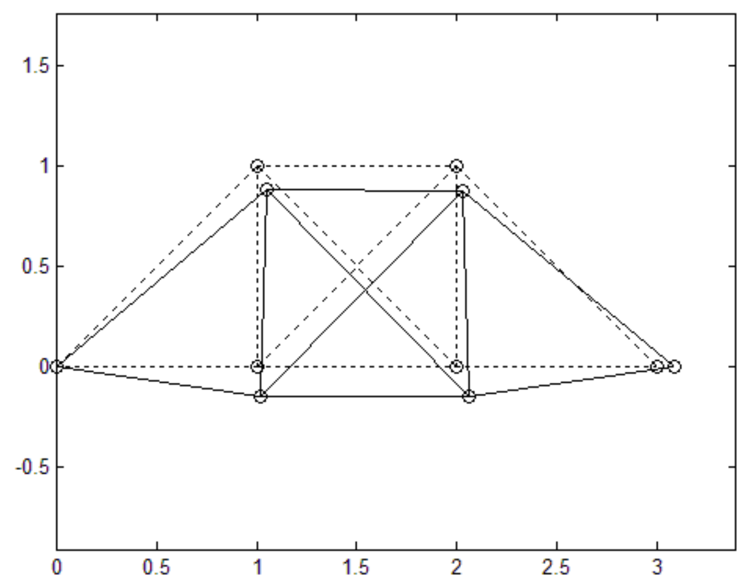
Courses Statics and strength of materials/Linear algebra:

Analysis of elastic truss frame

- Programming: from problem definition to code
- Manage large systems of equations,
- Visualize the stress distribution and deformations and optimization
- Introduction to FEM and Structural Mechanics



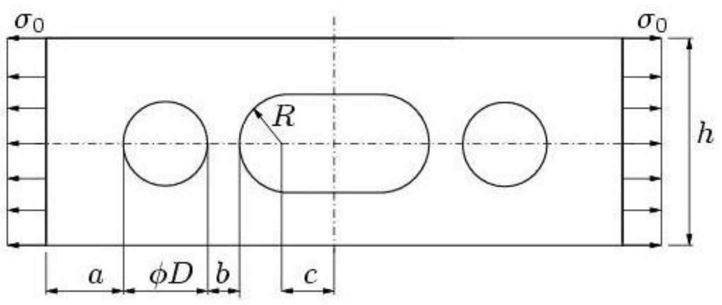


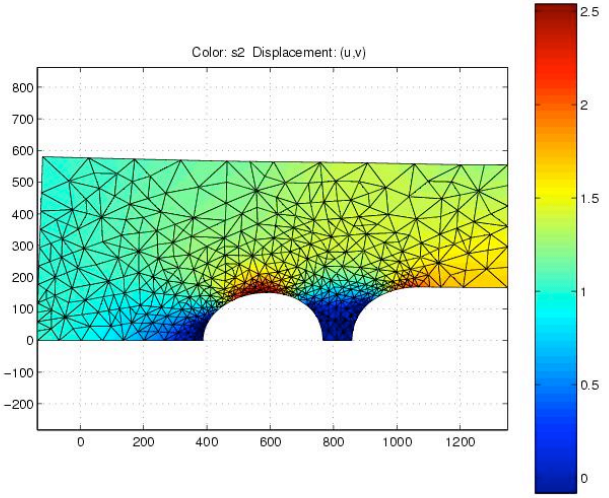


Courses: Mathematical Analysis in Several Variables and Solid Mechanics

Stress analysis of plane elastic plate with 3 holes

- Develop knowledge about stress distribution and how the stress is increased due to abrupt changes in geometry
- Skills to use the finite element method and introduction to error estimation and adaptive mesh refinement





EVALUATION AND RESULTS

- The computer is considered a valuable tool for match calculations and learning of mathematics
- Motivation to study, number of passed and students' general impression of math courses have increased. More training of math
- Programming skills have increased significantly
- More advanced and relevant analysis in product development projects
- Decision making is brought forward in the sense that students consider real systems and structures and solve real problem (reasoning and decision making at a higher level),
- Active learning is emphasized in simulations, open-ended problems and in the virtual/interactive learning environments that are used,

EVALUATION AND RESULTS

- The main goal that each student should gain knowledge, skills and ability to effectively use computational mathematical modeling and simulations in applications has been reached to a large extent
- Employers claim that the mechanical engineering students have became significantly better prepared for the managing and solving of open-ended problem, carrying out numerical simulations, programming and using modern industrial software.
- Teachers of advanced level courses verify that the students' ability to solve large complex problems has improved and that the computational skills in general are much better.
  - “Mechanical engineering students in average are much better prepared for the courses and can handle computations and projects more efficiently and at higher level compared to engineering students from other disciplines”

Questions?