First year engineering students' use and orchestration of resources to develop *actual student learning paths*: the cases of Calculus and Linear Algebra

Abstract

In this study we explore (1) which kinds of "resources" (material/traditional text/curriculum resource; human/social resources) engineering students used in two first year university mathematics courses (Calculus- CA; Linear Algebra- LA); and (2) how they used them for their learning of mathematics. In particular, using the theoretical lens of "resources" we investigated how students orchestrated the available resources to develop their own learning paths. Using a case study and mixed methods approach, we found that in the large CA course students followed various 'actual student learning paths', and they often found it difficult to choose from the large amount of resources on offer. Moreover, it appeared that in the large CA course students included human and social resources extensively, and this became a main part of their support system. At the same time, in the smaller LA course students could follow the learning path established by the lecturer/course developer. Although the mathematics was difficult to understand, students felt that they could follow a clear "path to success", something they were used to from their school times where the subject teacher provided the resources needed for students to pass their examinations. Furthermore, in this study we used, and illustrated the use of, a particular methodological tool: students' drawings of their resource system, which helped us to develop deeper insights into their learning strategies. Linked to this, and at the conceptual level, we coined the term 'actual student learning path' to describe students understandings of how their resource system was composed.

Introduction

In most western (engineering) universities students now have access to a plethora of resources, both digital/online resources and 'traditional' curriculum resources, such as textbooks, readers, worksheets, provided by the university and by lecturers. In particular in large first year courses (e.g. CA), students are often expected to use and blend the available resources according to their individual needs, to support their learning. The rationale for our study is that, as students now have an ever-increasing range of materials and an ever-widening range of formats to draw on to support their studies, their effective management of these resources is crucial. Moreover, in order to be able to understand (and develop) 'blended' mathematics learning, one has to know, which resources are used by students (from the ones on offer), and how they use them for their learning of mathematics. This, in turn, can inform the designers and teachers/lecturers of blended mathematics courses in their efforts to enhance the courses.

Drawing on recent research in university mathematics education, in a recent review study Biza, Giraldo, Hochmuth, Khakbaz, and Rasmussen (2016) have described the opportunities afforded by introductory university mathematics **textbooks**, and the actual use made of these curriculum materials by students. Anastasakis, Robinson, and Lerman (2017) investigated the **different types of tools** ('external' (to the university) and internally provided resources) that a cohort of second year engineering undergraduates used. Their results showed that, although to some extent students used resources external to their university, their practices were dominated by tools that their institution provided. The students in their sample chose certain tools mainly because these enabled them to pursue their examination-driven goals. The use of **visual resources** (e.g. online lectures) has been studied by Inglis, Palipana and Ward (2011). Using the Documentational Approach to Didactics Gueudet (2017) investigated mathematics teachers' interactions with resources at university, and Gueudet and Pepin (2018) explored how Brousseau's Didactic Contract can be interpreted, seen through the lens of (the use of) curriculum resources. In terms of LA, Grenier-Boley (2014) investigated LA tutorials: he noticed that the teacher's interventions and the student activity in class were linked to the kind of mathematical tasks chosen by the teacher from a list of exercises.

However, we note here that relatively little research is available on the broad range of resources available to first year university students to learn mathematics, and moreover how students actually orchestrate the resources in order to learn the mathematics. Typically, studies include the curriculum resources made available or recommended as part of mathematics courses, but there are also '**human resources**' (e.g. lecturers, tutors, peers) that students tap into, and **digital** and other resources mobilized by students themselves. In addition, we know little about the roles these various resources play in the intention of the lecturers (as compared to the students'). Inglis et al. (2011) suggest that students might need explicit guidance on how to combine the use of various resources into an effective learning strategy. Before this guidance can be given, or can be reified in a blended learning environment, more in-depth information on their actual use is needed. Hence, we ask the following **research questions**:

How do first year engineering students use and orchestrate the available resources for their learning of Calculus and Linear Algebra in their first year university, and which actual student learning paths can be identified?

Theoretical frames/literature review

The lens of "resources"

In this study we use the notion of "re-source/s" that students have access to and interact with in and for their learning. We assume that the ways university students learn mathematics is influenced/shaped by their use of the various resources at their disposal. By "use of resources" we denote, for example, which resources students choose (amongst the many on offer) and for what purpose (e.g. revision); the ways they align and orchestrate them (e.g. first lecture then checking the textbook); which ones seem central to achieve particular learning goals (e.g. for weekly course work, for examinations). However, we do not address the specific learning of CA and LA, that is how students interact with particular (e.g. cognitive) resources to learn particular topic areas in CA and/or in LA.

Gueudet and Pepin (2018) have defined student resources as anything likely to re-source ("to source again or differently") students' mathematical practice, leaning on Adler's (2000) definition of mathematics "re-sources" (in Adler's case used by teachers). In this study we distinguish between (1) material/traditional text/curriculum resource (including digital resources); and (2) human/social resources. As to (1) material/traditional text/curriculum resources: a further distinction has been made between (a) curriculum resources (those resources proposed to students and aligned with the course curriculum), and general resources (which students might find/access randomly on the web). Curriculum resources are typically developed, proposed and used by teachers and students for the learning (and teaching) of the course mathematics, inside and outside the classroom (Pepin & Gueudet, 2014). They may include text resources, such as textbooks, readers, and websites and computer software, and, for example, feedback on oral/video-/computer-based work. General resources are the non-curricular material resources mobilized by students, such as general websites (e.g. Wikipedia, YouTube). (2) In terms of human resources we refer to formal or casual human interactions, such as conversations with friends, peers or tutors.

In terms of distinction between the concept of *curriculum resources*, and that of other resources, in an earlier definition Pepin and Gueudet (2014) referred to *mathematics curriculum resources* as "all the resources that are developed and used by teachers and pupils in their interaction with mathematics in/for teaching and learning, inside and outside the classroom." Curriculum resources would thus include text resources (e.g. textbooks, teacher curricular guidelines, worksheets); other material resources, such as manipulatives and calculators; and web based/digital resources. For the term *blended learning* we adopt a pragmatic approach and define it as "using digital curriculum resources (Pepin, Choppin, Ruthven, & Sinclair 2017) in combination with other resources", whether they are traditional curriculum resources (e.g. textbooks), or human resources.

Actual student learning paths

The research literature in mathematics and science education shows many different terms and concepts linking to student learning paths, often associated with instructional theory and curriculum design. The following have been used in mathematics and science education: Hypothetical Learning Trajectory (HLT, Simon 1995); Local Instruction Theories (Gravemeijer 2004); Learning Trajectories/ Progressions (Lobato & Walters 2017); Learning trajectories in mathematics education (Weber, Walkington, & Mc Galliard 2015). Whereas some of the approaches focus primarily on learners (see first three in Lobator & Walters (2017), Simon's (1995) HLT approach includes instructional supports for learning and was originally conceived as part of a model of teachers' decision making. Simon introduced the term HTL to capture the result of a process in which a teacher posits a conjecture regarding his/her students' current understanding of a targeted concept (including potential challenges for them) and then develops learning activities that s/he thinks will support them in constructing more sophisticated ways of reasoning toward a particular learning goal. Simon and Tzur (2004) later highlighted the importance of and principles for selecting tasks that promote students' development of more sophisticated mathematical concepts. Building on this work, Clements and Sarama (2004) define learning trajectories as

> descriptions of children's thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks designed to

> engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain. (p. 83)

Whilst recognizing these important works, we regarded it necessary to coin a term that actually linked to our necessities: (1) when using the lens of resources to investigate students' learning paths, we looked at the alignment and orchestration of resources, not tasks or activities; (2) we investigated learning paths from the students' perspective, how they actually orchestrate∂ the resources for their own learning (and not how it was done by teachers/lecturers) and how they gave meaning to these self-created/orchestrated paths. We called these *Actual Student Learning Paths*.

The study

Using a case study approach, we explored two first year mathematics courses in a Dutch engineering university, CA and LA, as our cases. LA was taught to one group of approximately 130 students, mainly mathematics and physics students. CA was taught to approximately 2000 students (obligatory for all first-year students), in 6-7 groups of 300 students each (all types of engineering).

The CA course was organized with six hours of lectures and one hour of tutorials (in small groups of approximately eight students). It was also differentiated at three levels (A, B, and C), according to perceived level of difficulty and with varying level of emphasis on formal aspects of mathematics (e.g. proof). It appeared that the aims of the CA course were to provide students with a basic set of mathematical/computational tools they could subsequently use in their engineering studies and in their future work as engineers.

The LA course was organized with four hours of lecture per week, and three hours of tutorial (in groups of approximately 30 students). The LA learning aims were described as the acquisition of mathematical skills. Moreover, aims of the course were to help students develop the skills and realize the importance of correct mathematical communication, including writing formal proofs. Completing a mathematical writing assignment was part of the course requirements to reach this aim. It appeared that the purpose of LA was to prepare students for higher mathematics (used in the mathematics and physics courses).

In terms of *participants*: in total 24 students participated in the study: 18 CA students (involved in nine different engineering programs and all taking the B level CA course); 1 CA student who dropped out of university; 5 LA students (all studying for the 'applied mathematics' engineering course). In terms of background, of the interviewed CA students 15 came from secondary schools in the Netherlands, three came from other (international) educational systems.

For this paper we used data from the following data collection strategies for analysis:

- individual and focus group interviews with students (24): 19 CA (incl. 1 drop out) + 5 LA

- students' drawings (see example Figure 1): students were asked to draw Schematic Representation of Resource System (SRRS - see Pepin, Xu, Trouche, & Wang, 2017), and during the interviews students were asked to explain their resource use based on their SRRSs.
- analysis of documents/curriculum materials provided by the university for the students (e.g. examples of past examinations, syllabi, reader/s, textbook/s, video clips, videos of the lectures).

In terms of analysis, the interviews were transcribed and student interviews were analyzed with the help of the ATLAS-ti software. Interview quotations were coded with codes based on the themes from the literature with reference to the use of different resources, and to student approaches to learning mathematics, and then 'constantly compared' with those themes mentioned by students. In the next step student drawings of the *actual student learning paths* were compared with student explanations, what students said and how they explained their learning (and their resource systems), in order to identify patterns of those learning paths.

Results

Overall, students used different/additional resources in the two courses, and they used the available resources differently for LA than for CA: (1) Basically, all LA curriculum resources offered/provided were used, and students worked with them according to the lecturer's guidance. (2) The CA resources seemed to be a large bag of 'tools', a pile of 'bricks', that the students could pick from (according to their needs) and use for their learning. However, how to build an actual learning path for the learning of CA was not clear. These differences appeared to be related to (a) the size and student audience of the courses (130 students in LA; 2000 in CA), and this, in turn, was connected to different organizations of the courses (4 hours lecture and 3 hours tutorials in LA; 6 hours lecture and 1 hour of tutorial in CA); and (b) the organization and alignment of the resources with the assessment/tests. For example, there was a clear intended (by the lecturer) learning path in LA, with exercises aligned with the examinations. Students mentioned that if they worked according to/with the reader and did "all exercises in the reader" (plus of course the weekly assignments), they would pass the examination. In the CA course however, a large number of support tools were proposed (e.g. on the web, in print), with huge number of exercises, tasks, etc. that were not clearly aligned with the end examination. Students said that it was not possible to do all exercises, read all materials provided, and they often had difficulties choosing from the immensity of resources provided.

Using the lens of resources, more precisely we used students' drawings of their resource systems, to identify students' actual learning paths, that is which resources they used and how they orchestrated them for their learning. From these drawings and the accompanying interviews we could identify particular learning paths and different combinations of resource use. The learning path of the LA course appeared to be a traditional, which all students followed, where students could identify 'core resources' (e.g. the reader, past examinations, weekly tests), and where a particular 'blending of resources' was recommended. This would help students to understand the weekly coursework and to pass the final examination. In addition, students had more help from the tutors and lecturer (which used to be their experience at secondary school).

In contrast, the students on the CA course outlined several learning paths, based on their individual experiences, and for each path different resources came into play/were blended. For example, and in contrast to our survey results (where the lecture was "ticked" as one of the main resources; see Kock & Pepin, 2018), in their drawings and in the interviews only two students put the lecture as a center point for their learning. It appeared that the lecture was for information, what had to be learnt, rather than for the actual learning.

"If I hear them talk about it, it's easier for me to revise/practice when I've already seen it, heard about it." (see Joanna's drawing/writing) below, Figure 1)

At the same time, either the lecture or lecture notes were mentioned by all students as a supporting resource for their learning of CA. Interestingly, a large number of students pointed to human resources, in particular their friends and peers, and the tutor, as main resources. As perhaps expected, books and tests/quizzes/exercises were mentioned as a huge help. Here the digital resources (e.g. You tube; Khan Academy) seemed to gain importance. Altogether, the CA learning paths showed a complex picture of students using a mixture and ever-increasing number of external resources (in particular human and digital nature). Due to a (re-) analysis of the SRRSs in relation to the interviews, we could identify a small number of 'non-traditional' but productive *actual student learning paths* that students seemed to develop for themselves, which we claim should be considered (by lecturers/course developers) when aiming to "blend" learning in a course such as the large CA course. (Examples and more explanations will be provided in the presentation at the conference.)

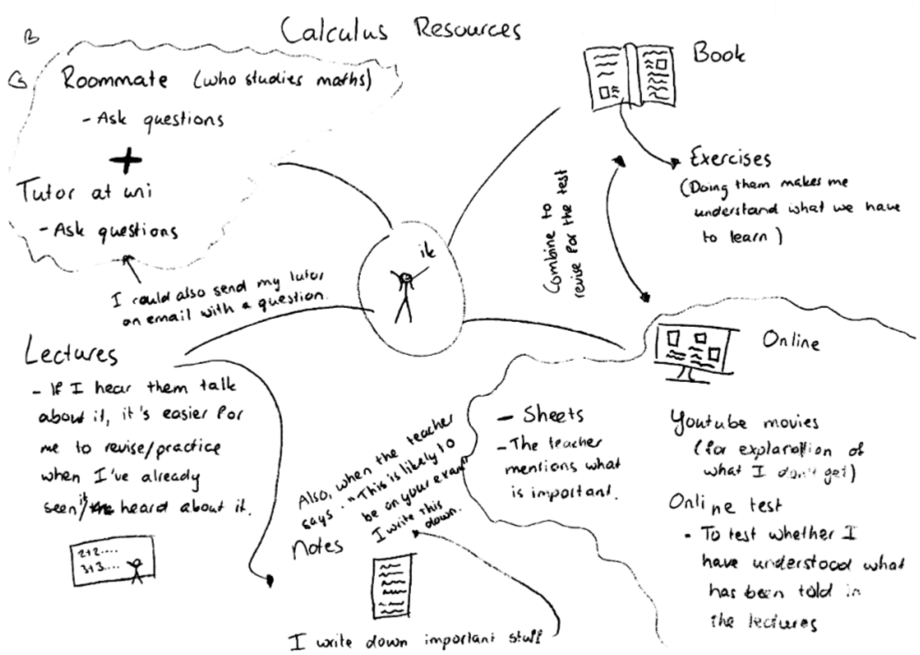

Figure 1 Joanna's drawing of her resource system

Conclusions

Based on our results we claim that it is not sufficient to provide a plethora of curriculum resources, may they be digital, traditional text or human resources, but that serious consideration should be given to how students might combine these resources, orchestrate them into their preferred individual learning paths. In addition, it is advisable to help, perhaps even to train students how to develop such learning paths, and these might be different from one subject to another (even from one mathematics course to another), in order that students become more effective in their resource use.

This, we claim, is the responsibility of the lecturer/teacher/course designer. Such course development would involve purposeful design, including the development/envisaging of particular (intended) learning paths, possibly considering selected *actual student learning paths*, and the design of particular resources supporting such paths. Simply providing access to curriculum resources may not help students to 'blend their learning' and orchestrate the resources on offer, but may rather confuse and overwhelm them (due to the immensity of resources on offer), and drive them towards "learning for the test". As Anastasakis, Robinson, and Lerman (2017) claim, under such conditions "students use the most popular resources [and] they aim mostly for exam-related goals" and use "certain tools because these enable them to pursue their exam-driven goals" (p. 67).

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