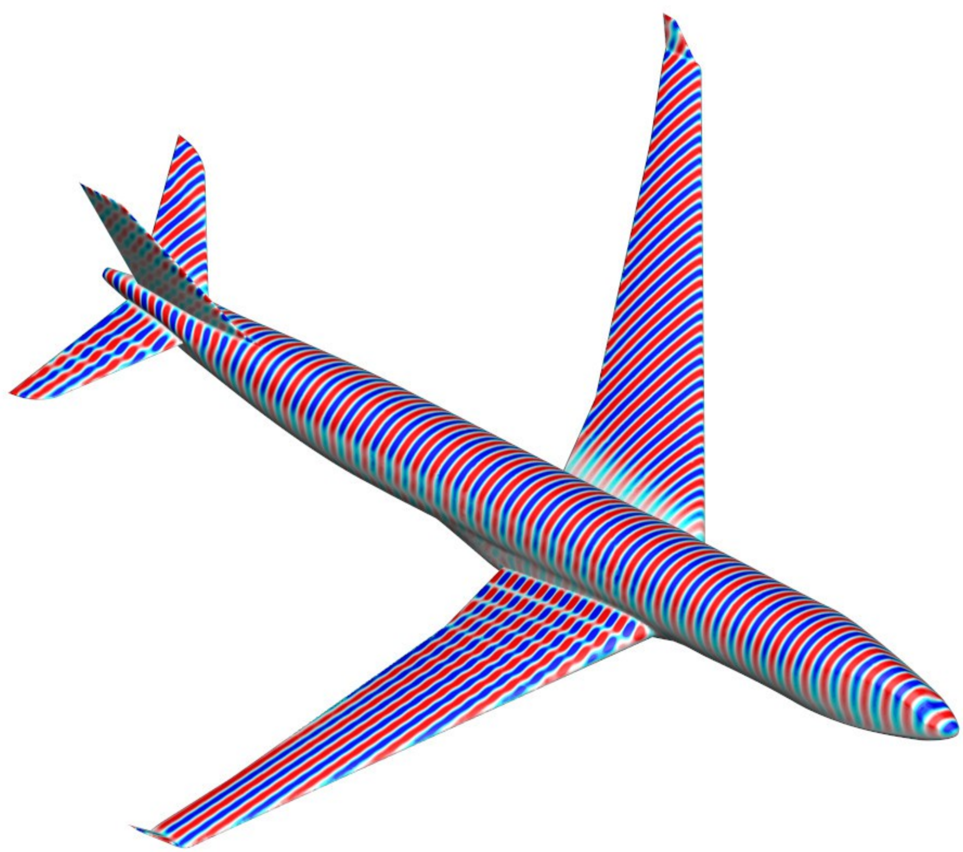
# INT13

# An integral approach to electromagnetic scattering:
# A Workshop on Recent developments in Theory and Applications

**National Aerospace Laboratory NLR**

**Amsterdam**

**Tuesday 12 November 2013**



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# Introduction

Integral equations are very attractive for modeling electromagnetic scattering, because their solution intrinsically has the correct asymptotic far field behavior and lacks dispersion errors that trouble solutions obtained with models based on differential formulations. Although generally formulated for the frequency domain, scatterers with a nonlinear response require modeling in the time domain.

Discretisation of *boundary* integral equations leads to a numerical model with very favorable scaling properties. With the advent of accelerating techniques for frequency (e.g. Multi Level Fast Multipole Algorithm) and time domain (e.g. Plane Wave Time Domain) the electrical size of the problems that can be addressed is nearly unlimited on modern hardware.

In the frequency domain, discretisation and solution methods have reached a technology readiness level that allows these to be applied on a daily basis in an industrial environment. The development of time-domain techniques is advancing fast, but still lags behind.

However, many questions remain open and therefore both are active areas of research. For the frequency domain the main research topic is how to improve efficient solution of the system of linear equations that results after discretisation, while in the time-domain focus is on obtaining more insight in the cause of so-called *late-time instabilities*. Many research groups are simultaneously active in both fields.

The purpose of the proposed workshop was three-fold. First of all we had the intention to provide a forum for researchers in the field of time and frequency domain integral equations to exchange new ideas and start new cooperations, initiated by a number of invited talks by leading experts. Furthermore, the workshop would provide an opportunity to PhD students to share the results of their research with more experienced researchers, get feedback and extend their network.

Finally, we organized a common exercise prior to the workshop, in which all attendees of the workshop could participate. In this common exercise four challenging test cases for time-domain integral equation solvers have been presented. We have collected the results in advance and one of the organizers presented these at the workshop. This helped understand the benefits and shortcomings of the different approaches to formulate time-domain scattering modeling through integral equations in use today.

# Organisation

***Organizers:***

Dr. Harmen van der Ven, National Aerospace Laboratory NLR

Dr. ir. Duncan R. van der Heul, Delft University of Technology

Dr. Kristof Cools, George Green Institute for Electromagnetics Research, University of Nottingham

***Committee of recommendation:***

Prof. dr. ir. Jaap van der Vegt, Twente University

Prof. dr. Will Schilders, Eindhoven University of Technology

Prof. dr. ir. Kees Vuik, Delft University of Technology

## Participation

We were very pleased to welcome a relatively large group of participants to the workshop, both from academia and industry. Due to the fact that we found one of the leading experts in the field of integral equations, Prof. Michielsen, willing to accept our invitation to be the main speaker at the workshop, we even had a number of participants from abroad: France, Germany and the United Kingdom.

The distribution of the participants is listed in Table 1 and visualized in Illustration 1.

| Country | Number of participants |
|---|---|
| Netherlands | 22 |
| United Kingdom | 6 |
| Belgium | 2 |
| Germany | 2 |
| France | 1 |
| United States | 1 |

*Table 1: Number of participants according to domicile.*

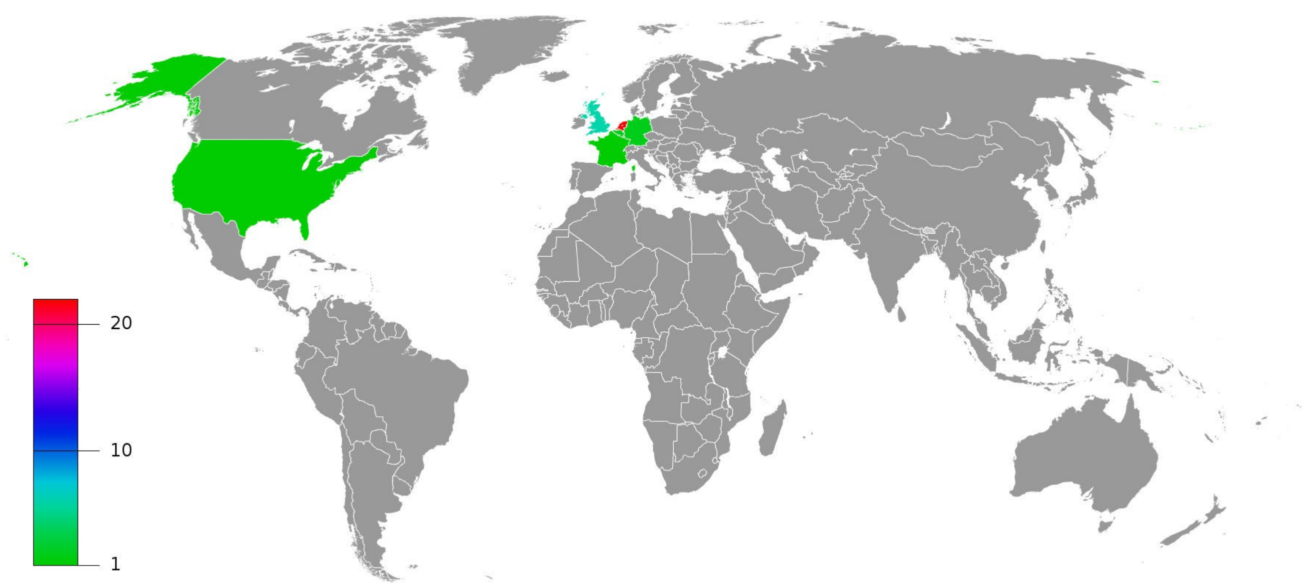


*Illustration 1: The location of the home institutions of the participants of the workshop.*

# Program

## *Overview*

The morning session of the workshop was directed mainly to time-domain formulations of boundary integral equations, while after lunch the focus moved to the frequency domain. Apart from the invited speakers a number of PhD students presented their research, focused on or strongly related to boundary integral equation models for electromagnetic and acoustic problems. The scientific part of the workshop was concluded by the main speaker. Professor Michielsen gave a very interesting presentation on fast direct solution methods for the linear system of equations that results from discretisation of the Electric Field Integral Equation in the frequency domain. The abstracts of the di

## *Common exercise*

As an intermezzo between the different talks, Kristof Cools presented the results of the common exercise for numerical methods for electromagnetic scattering simulations based on the discretisation of time domain integral equations. Unfortunately, the busy schedule of the attendants of the workshop resulted in only two parties participating in this project. However, it was very inspiring and reassuring that the two participating groups, The George Green Institute for Electromagnetic Research and NLR/TUDelft had nearly coinciding results for the different test cases. This clearly shows that models based on these techniques are ready to be used in an industrial setting.

## *Abstracts of the talks*

**Kristof Cools, George Green Institute for Electromagnetic Research**
***Stable and Accurate Time Domain Boundary Element Methods***

In recent years, time domain boundary element methods have become increasingly popular. Like their frequency domain counterparts, time domain boundary element methods are well suited to model scattering and transmission of electromagnetic waves at piecewise homogeneous components that can radiate in an unbounded environment. Moreover, being time domain methods, they deliver broad band results in a single simulation and they can be coupled to non-linear systems such as digital circuits. In recent years, the stability, and accuracy of time domain boundary element methods have been significantly improved upon. In this presentation, an overview will be given of the most common approaches to time domain boundary element methods and it will be elucidated how these approaches relate to the schemes' accuracy and stability. Special attention will be devoted to the recently developed class of space-time Galerkin schemes, which have improved the methods accuracy even further.

**Lehel Banjai, School of Mathematical and Computer Sciences, Heriot-Watt University**
***Convolution quadrature for time-domain boundary integral equations***

We will describe convolution quadrature (CQ) as a discretization method for time-domain boundary integral equations of acoustics and electromagnetism. Advantages of CQ in terms of stability and ease of implementation will be mentioned. Furthermore a new result on a positivity preservation property, important for coupling FEM and BEM in time domain, will be discussed. Finally the dissipative/dispersive nature of CQ for wave propagation will be addressed, with new and old remedies presented. The results will be illustrated by numerical experiments on model problems.

**Elwin van 't Wout, National Aerospace Laboratory NLR/Delft University of Technology**
***Stability Analysis of the Time Domain Integral Equation Method for Radar Scattering Analysis***

The applicability of Time Domain Integral Equation methods has been limited due to the persistent occurrence of instabilities. In this presentation, we will use a stability theorem for a specific variational formulation of the Electric Field Integral Equation. The Sobolev spaces in the functional framework will be adapted to the differentiated version of the EFIE, which can be discretized with more efficient schemes. The most efficient temporal discretization method is the Marching-on-in-Time scheme that is based on collocation. For the stability theorem to be applicable to collocation schemes, a discrete equivalence with the space-time Galerkin scheme has to be derived. This results in the Marching-on-in-Time scheme with quadratic spline basis functions that fit within the functional framework of the stability theorem. Computational experiments confirm the stability of this specific TDIE method.

**Yves Beghein, Department of Information Technology, Gent University**
***Higher-order temporal discretization schemes for time domain boundary element methods***

Existing temporal discretization schemes for time domain boundary element methods (TD-BEMs) are usually classified as either collocation or Galerkin methods. When piecewise polynomial basis and testing functions are employed, both approaches are intimately related. The advantage of the Galerkin formulation, however, is that its extension to higher order is similar to that of other finite element schemes. Thus, both h-refinement (decreasing the time step) and p-refinement (increasing the polynomial order) can be used to control the accuracy of TD-BEM simulations. In this talk, the extension to higher order is discussed for various types of boundary integral equations. The superior performance of p-refinement is illustrated using numerical examples.

**Ellaheh Barzegar, Department of Electrical Engineering, Eindhoven University of Technology**
***Wave propagation in a 1D slab with stochastic properties***

Wave propagation in and through a 1-dimensional dielectric slab with stochastic properties is studied as a simplified example of a broader class of wave propagation in random media. This problem can be formulated in terms of a 1-dimensional domain-integral equation in the frequency domain. For a deterministic setup, the integral equation admits an efficient matrix-vector product, owing to the properties of the Green's function and the local field-material interactions. However, when the slab has stochastic permittivity, the problem becomes nonlinear with respect to the parameters that describe the stochastic nature of the problem, which makes the full characterization of the solution to the problem more intricate. Instead of resorting to a sampling approach, like Monte-Carlo methods or stochastic collocation, the possibilities of applying intrusive polynomial chaos are investigated and the implications for the corresponding numerical scheme are addressed, as well as the conditioning and efficiency of the scheme.

**Bruno Carpentieri, University of Groningen**

***Preconditioned Krylov subspace methods for solving high-frequency cavity problems in electromagnetics***

The numerical solution of Maxwell's equations in large unbounded domains may be carried out using the boundary element method, which reformulates Maxwell's equations as a set of integral equations defined only on the surface of the scattering body, and then solves them for the electric and magnetic currents distributed on the surface of the object. For geometries with cavities, the exterior Maxwell's problem may be described in the frequency domain by the following variational problem: find the surface current $\vec{j}$ such that for all tangential test functions $\vec{j}_t$, we have

$$\int\int G(|y-x|)\left(\vec{j}(x)\cdot\vec{j}^t(y)-\frac{1}{k^2}\left(\nabla_\Gamma\cdot\vec{j}(x)\right)\cdot\left(\nabla_\Gamma\cdot\vec{j}^t(y)\right)\right)dxdy=\frac{i}{kZ_0}\int\vec{E}_{inc}(x)\cdot\vec{j}^t(x)\,dx \quad (1)$$

In Eqn.(1), known as the Electric Field Integral Equation, we denote by $G(|y-x|)=e^{ik|y-x|}\cdot(4\pi|y-x|)^{-1}$ the Green's function, $k$ is the wavenumber and $Z_0=\sqrt{(\mu_0\epsilon_0)}$ the characteristic impedance of vacuum ( $\epsilon$ is the electric permittivity and $\mu$ the magnetic permeability). On discretizing Eqn.(1) in space by the standard Method of Moments, and expanding the surface current $\vec{j}$ into the set of Rao-Wilton-Glisson basis functions, we are led to the solution of extremely large and tough systems of linear equations, which is often the bottleneck of the computation in large-scale scattering analysis, e.g. the radar-cross-section calculation of realistic targets in aerospace simulations.

In this talk, we present our recent advances in the design of preconditioned Krylov methods for solving cavity scattering applications. We discuss various numerical linear algebra aspects, that are the choice of the iterative method, the characteristics and performance of the parallel Multilevel Fast Multipole Algorithm (MLFMA) that we used in our experiments, the design of multilevel preconditioners combined with MLFMA. We also consider symmetry-preserving strategies both for the iterative method and for the preconditioner. These numerical linear algebra tools have enabled us solving very large scattering problems efficiently on a moderate number of processors.

**Martijn van Beurden, Department of Electrical Engineering,**

**Eindhoven University of Technology**

***A spectral volume integral equation approach for periodic dielectric media***

Scattering by media with a periodic repetition is a very well-studied subject with relevance to modern applications like absorbers for anechoic chambers, meta-materials, electromagnetic band gap materials, optical gratings, nano-wires, solar cells etc. In most of these applications, periodicity in two directions is observed in combination with a third aperiodic direction. A further simplification that is assumed here is that all the media are dielectric in nature, i.e. the magnetic properties are assumed to be homogeneous throughout.

Owing to the periodicity of the problem, the analysis can be restricted to a unit cell. For a homogeneous background, a volume integral equation in the frequency domain can be formulated where the kernel of the integral equation can be represented both in the spatial and in the spectral domain. When the periodic setup is embedded in a layered dielectric medium, a closed-form expression of the integral kernel exists only in the spectral domain. The corresponding volume integral equation is then discretized in the spectral domain, which leads to an infinite system of coupled one-dimensional integral equations, where the coupling originates from the inhomogeneity of the permittivity. Particular attention must be paid to the field-permittivity interaction, such that an efficient matrix-vector product is obtained with which the resulting linear system can be solved iteratively, without sacrificing the accuracy of the method. Further, the one-dimensional integral equations have both a convolutional and a semi- separable structure, either of which can be exploited.

**Tobias Rienmüller, ZeTeM, Zentrum fur Technomathematik, Universität Bremen.**

***A Spectral Volumetric Integral Equation Method for Ocean Acoustics with Depth-Dependent Background Sound***

We present fundamental ingredients for the numerical analysis for a spectral volumetric integral equation method for ocean acoustics with depth-dependent background sound speed. The scattering of time-harmonic acoustic waves is modeled by the Helmholtz equation with Dirichlet boundary condition on the ocean surface and Neumann boundary condition on the bottom of the ocean. However, reasonable models for sound propagation over large distances in the ocean imperatively require a depth-dependent background sound speed. In particular, we introduce a Lippmann-Schwinger equation, that is equivalent to our scattering problem and a suitable radiation condition.

**Nicolas Salles, Department of Mathematics, University College London**

***The Method of Reduction of the Dimension for the evaluation of singular and near singular integrals arising in 3-D Electromagnetic Scattering problems.***

The accurate evaluation of integrals with singular integrand arising in Galerkin Boundary Element Methods can be a problem. Furthermore, to obtain a given accuracy, the number of integration points depends on the geometry of the elements of the mesh for most of the numerical techniques. We present a new method to evaluate explicitly integrals of the singular part of the kernel. This method reduces an integral on an m-dimensional domain into a linear combination of integrals on (m-1)-dimensional domains and finally we obtain 1-D integrals which can be evaluated numerically or even explicitly. Our method relies upon the homogeneity of the integrand but it can be used to reduce integrals of the fundamental solution of the 3-D Helmholtz equation which is not homogeneous but is an analytical function. We will introduce this method and show several significant results for the 3-D Helmholtz equation with constant and linear basis functions. Finally, we will talk about its application to electromagnetic problems (EFIE and MFIE) with low order RWG basis functions.

**Gabriele Gradoni, School of Mathematical Sciences, University of Nottingham**
***Characterizing Electromagnetic Fields of Integrated Electronic Systems in Enclosures: a Ray-Wave Approach***

In this presentation, we describe how field-field correlation functions may be efficiently propagated using ideas based on ray propagation. The key point is to make a connection between the field-field correlation function and a corresponding Wigner function, which is obtained from it by Fourier transformation. The Wigner function underlying this approach originated in quantum mechanics, where it can be used to make an analogy between the evolution of the wave function and the evolution in phase space of the underlying classical trajectories. In the context of EM problems, the Wigner-function approach has been championed by Marcuvitz using the quasi-particle picture of wave evolution (N. Marcuvitz, Proc. IEEE, 79-10, 1991). In the approach we propose, to approximate the propagation of field-field correlation functions by propagating phase-space densities of ray families, which is effectively a lower-dimensional calculation and therefore easier to compute. As an example, we consider a model problem in which the field-field propagator is evaluated in free space. The solution works from the near- to the far-field and serves as a proof-of-principle for propagation in more complex environments. In particular, the Wigner-function approach can be extended to boundary-value problems by using the results of the random matrix theory. We show that this philosophy has common aspects with that adopted in the formulation of the Maryland Random Coupling Model for complex electromagnetic enclosures.

**Prof. Eric Michielssen, Radiation Laboratory, Electrical Engineering and Computer Science Department, University of Michigan**
***A Parallel MLMDA-Based Direct Integral Equation Solver***

Fast direct integral equations methods for analyzing time-harmonic scattering phenomena constitute an active area of research in the CEM community. These solvers present an interesting alternative to FMM-based iterative schemes for problems that are inherently ill-conditioned and/or involve many excitations. Present fast direct methods leverage the low-rank (LR) nature of blocks of the LU factors or inverse of the method of moment (MoM) matrix (if and when this property manifests itself). In essence, they leverage the limited number of degrees of freedom of the fields radiated by sources in the presence of the scatterer (or portions thereof), observed across a finite region of the scatterer (whenever the concept makes sense). LR compression strategies do not lead to low-complexity solvers for electrically large concave structures. This fact was demonstrated in [H. Guo et al., Antennas and Wireless Propagation Letters, IEEE, vol. 11, 2012], where blocks of LU factors that proved incompressible by LR methods were re-compressed by MLMDA/butterfly methods. Somewhat unexpectedly, this study showed that MLMDA/butterfly scheme produced compression above and beyond that provided by LR schemes, requiring only O(N log^2 N) final storage irrespective of the nature or electrical size of the scatterer. The practical applicability of the scheme however was severely hindered by the fact that the MLMDA/butterfly scheme acted as an add-on to a LR solver.
Here, we report on a new direct integral equation solver by that utilizes MLMDA/butterfly schemes for compressing the LU factors of an MoM matrix. When compared to its predecessor, the new solver has three important features of note.

- The solver entirely bypasses the LR compression step of its predecessor and operates directly on butterfly-compressed blocks. The latter is achieved using new schemes for rapidly adding and multiplying butterfly-compressed operators.

- The solver executes in parallel, using a hybrid OpenMP-MPI strategy to accelerate the hierarchical inverse/decomposition of the MoM matrix. Despite the inherently sequential nature of this operation, the solver exhibits excellent scaling properties up to several hundred processors.
- The solver applies to large-scale 3D (as opposed to 2D) analysis and was implemented to invert a combined field (as opposed to an electric field) integral operator. The solver is capable of inverting MoM matrices that discretize combined field integral equations modeling scattering from electrically large structures involving millions of unknowns on a small cluster using just a few hours of CPU time.

## Outcome of the meeting

Frequency domain boundary  integral equation methods are already widely applied to solve electromagnetic scattering problems. Especially since the introduction of acceleration techniques like the Multilevel Fast Multipole Algorithm, very large-scale scatterers can be efficiently analyzed due to the favorable scaling properties of discretized boundary integral equation methods.
The good agreement between the results obtained by the time-domain integral equation methods in the common exercise presented at the workshop shows that also time-domain methods have made a leap in technology readiness level over the last few years. To quote one of the speakers: “Performing computations with a time-domain integral equation methods is now a deterministic process”.
The interest in the workshop shows that many researchers are involved in the the further development of boundary and volume integral equation methods. Both spatial and temporal discretisation techniques and solution methods for the resulting linear systems are still being improved and optimized.
We expect that the workshop will lead to more intensive cooperation between the different research groups in the Netherlands, who are working on integral-equation-based numerical methods and that more meetings will be organized in the near future.

We would like our sponsors NWO, NDNS+, 3TU.AMI, TU Delft and the National Aerospace Laboratory NLR for their generous support to make it possible for us to organize this event and in particular the NLR staff for their hospitality.

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