Solution

The correct answer is: 9.

To create an admissible new enclosure, it helps to look at the points where different enclosures meet in the interior. For instance, look at the point marked B in Figure 22 below.

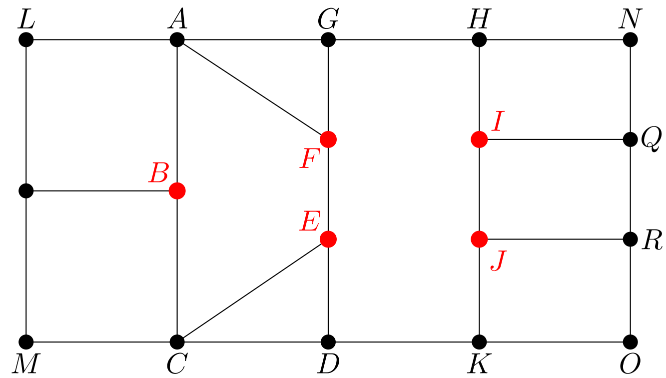


Figure 22: Santa's original stable. The problem points B, E, F, I, J are marked in red.

In B, three enclosures meet. If we try to put reindeer in the empty enclosures surrounding B, you immediately run into a problem: for instance, suppose we were to put a red-nosed reindeer in the large enclosure to the right of B. Since that enclosure shares a wall with the two enclosures to the left of B, we must put brown-nosed reindeer in those two enclosures to prevent the red-nosed reindeer from fighting. But the two enclosures to the left of B share a wall, leading to a conflict between the two brown-nosed reindeer.

If however four enclosures met in this point, then there would be no problem: going clockwise around the point, assign red-nosed and brown-nosed reindeer to the enclosures alternately. This pattern repeats; whenever an even number of enclosures meet in one internal point, that is, one of the points B, E, F, I, J, we can assign reindeer to those enclosures without risking a conflict. A valid solution is then exactly a division where all internal points are surrounded by an *even* number of enclosures.

We can easily find new divisions by using this property. The next challenge is to make sure we count all distinct possibilities, and do not double-count any. To do so, we proceed systematically. Note that there are exactly five points (B, E, F, I, J) in the interior where an odd number of enclosures meet; we will call these the *problem points*. We must affix at least one wall to each of these problem points.

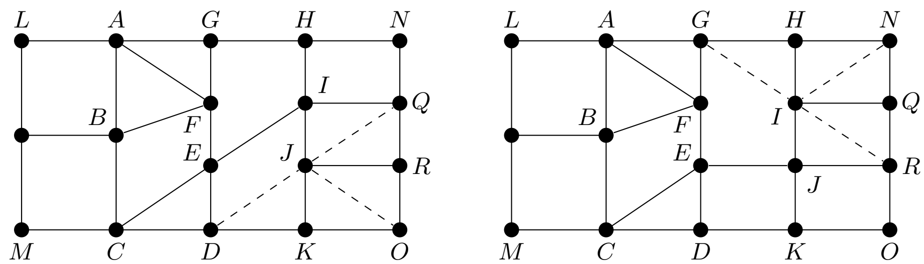
Since there are five such points, and we may only build three walls, each problem point must get exactly 1 additional wall. To see this, note that we cannot affix two walls to one problem point, since this would turn it again into a problem point (odd + 2 = odd). But if we affix three walls to one problem point, then the remaining endpoints of these walls can fix at most three additional points, fixing at most four of the five problem points.

Consider the problem point B. We can fix it in four distinct ways, namely by drawing exactly one of the lines BF, BE, BL, and BM. We will only consider the cases where we draw BF and BL, since, by symmetry, the cases BE and BM will yield additionally the same number of possibilities.

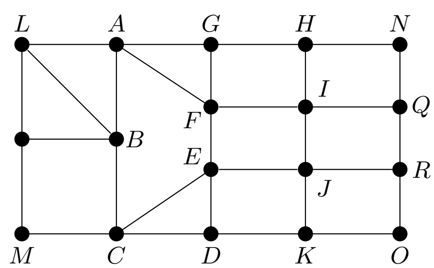
Case BF: After drawing BF, the remaining problem points are E, I and J. We can draw two more lines, so one of these lines must be used to connect two points among $\{E, I, J\}$. This can be done in two ways: EI, EJ.

Consider the first the case where we draw EI. Then J is the only problem point that still needs to be fixed. There are three options for the final line that fix problem point J: JD, JO, and JQ.

Next, consider the case where we draw EJ. Then we are again left with three options for the final line: IG, IN, and IR.



Case BL: After drawing BL, there are again two lines left to draw. This time, there are four problem points left, so we must use the remaining lines to connect up these points. There is only one valid way to connect these points, which is to connect IF and EJ.



These two cases together yield seven solutions. As mentioned previously, the cases where we draw BE and BM will yield again seven solutions by symmetry. Therefore, the total number of solutions is fourteen.