Engineering students' experience of vector analysis: a case for focus on the dot product

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(This research conducted at the University of Cape Town)

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Background and context

- 2015-2017; 6 semester cohorts
- Dynamics students (mostly mechanical engineering)
- Data shown here for first 4 cohorts (N = 325)
- Vector proficiency test; 29-31 items
- 8 items involve scalar (dot) product
- Computation: 2 easiest items
- Use in a context: amongst most difficult; includes 4 most difficult

Rasch analysis

- Premise: student response to item is a function of proficiency and item difficulty. Emphasis on *measurement*.
- Problems ranked on axis of difficulty, matched against axis of student proficiency (the item map)
- Item characteristic curves show observed versus expected responses of quantiles
- Multiple choice distractor curves show distractor preference across quantiles
- Software used: RUMM2030

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---------------------------------------------------------------------------------------
LOCATION          PERSONS      ITEMS [uncentralised thresholds]
---------------------------------------------------------------------------------------
  5.0                        |
                             |
                             |
                             |
                           × |
  4.0                        |
                           × |
                             |
                             |
                          ×× |
  3.0                      × |
                         ××× |
                    ×××××××× |  I0013.1
                          ×× |  I0023.1
                      ×××××× |
  2.0              ××××××××× |  I0022.1  I0006.1
                    ×××××××× |
                 ××××××××××× |  I0038.1
            ×××××××××××××××× |  I0040.1  I0034.1
              ×××××××××××××× |  I0007.1  I0037.1
  1.0       ×××××××××××××××× |  I0021.1
                  ×××××××××× |  I0016.1  I0020.1
                ×××××××××××× |  I0005.1  I0024.1
                  ×××××××××× |  I0035.1  I0029.1
                  ×××××××××× |  I0008.1  I0039.1  I0002.1
  0.0                ××××××× |  I0017.1  I0028.1
                    ×××××××× |  I0031.1  I0026.1
                       ××××× |
                          ×× |  I0009.1  I0019.1  I0033.1
                         ××× |  I0003.1  I0025.1
 -1.0                    ××× |
                          ×× |  I0004.1  I0010.1
                           × |  I0015.1
                             |  I0011.1
                           × |  I0012.1
 -2.0                        |  I0036.1
                             |
                             |  I0018.1
                             |
                             |  I0027.1
 -3.0                        |
                             |  I0014.1  I0001.1
                             |
                             |
                             |
 -4.0                        |
---------------------------------------------------------------------------------------
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Items requiring use of the scalar product as part of a larger process (23, 22, 13, 6, 7, 21)

Items requiring basic scalar product (1, 14)

Two computation items

1. Evaluate the dot product $\langle 5,1,1\rangle \cdot \langle 3,7,-1\rangle$.

(A) 32 (B) 16 (C) 63 (D) 21 (E) 50

14. Evaluate the dot product of $8i-j+4k$ and $3i+6j-2k$.

(A) 10 (B) 24 (C) 11 (D) 25 (E) 38

Six contextual items

13. Angle between student-defined vectors.

23. Resolve a given vector into components parallel and orthogonal to another given vector.

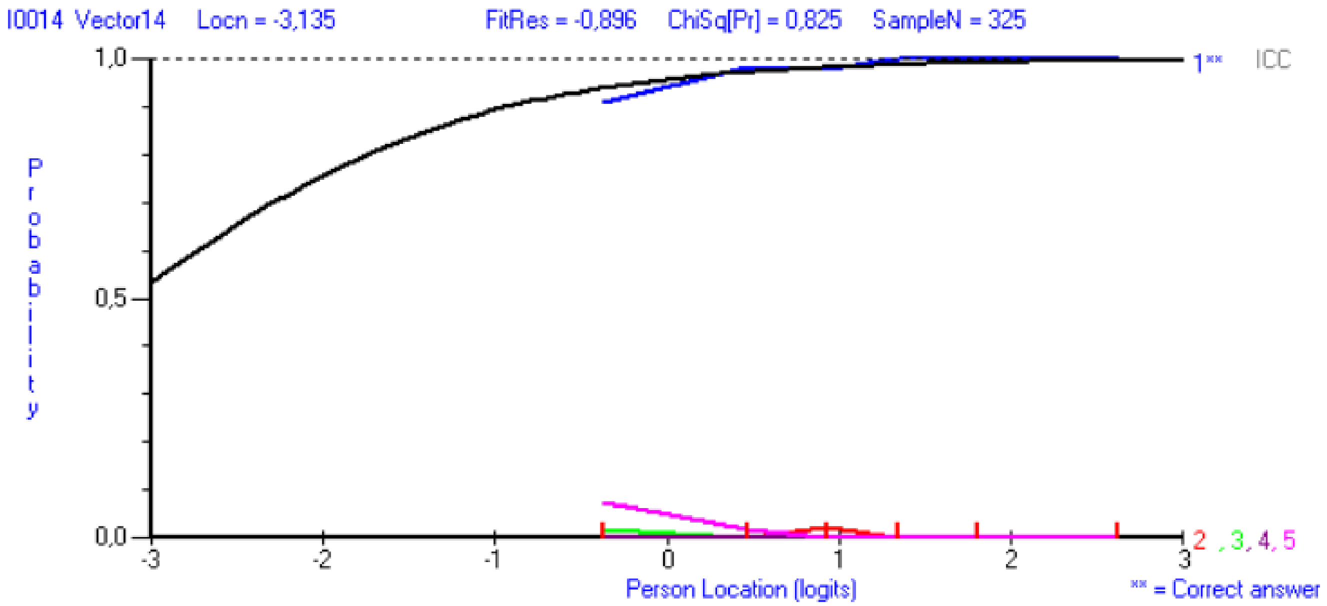
22. Angle between student-defined vectors.

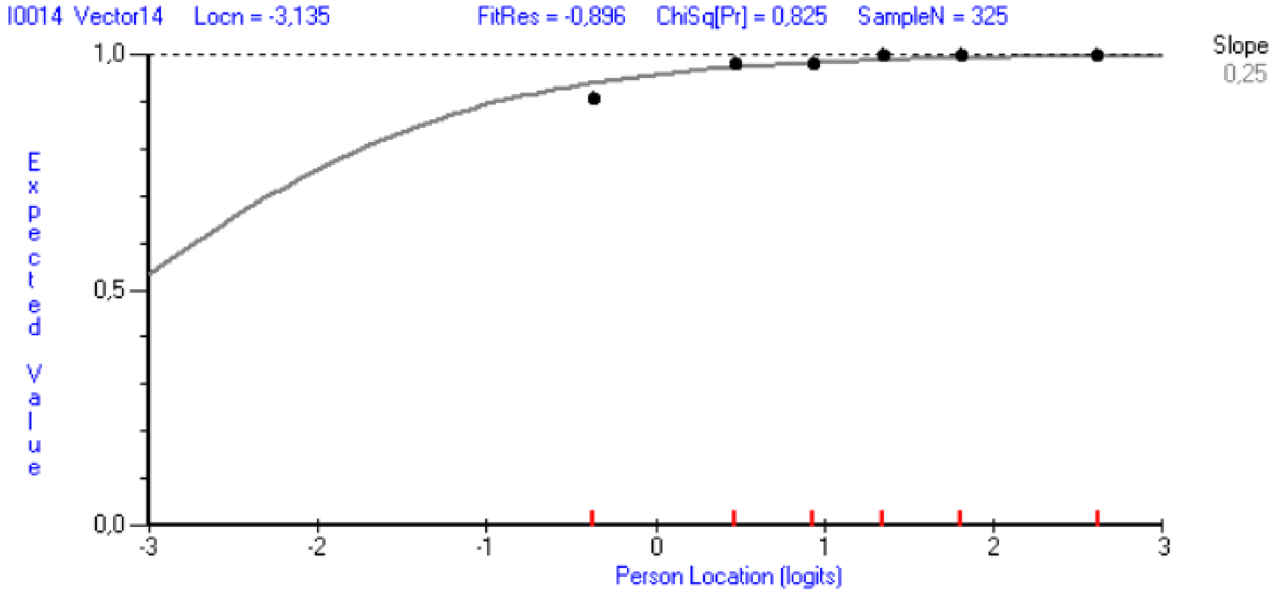
6. Resolve a given vector into components parallel and orthogonal to another given vector.

7. Volume of parallelepiped (scalar triple product).

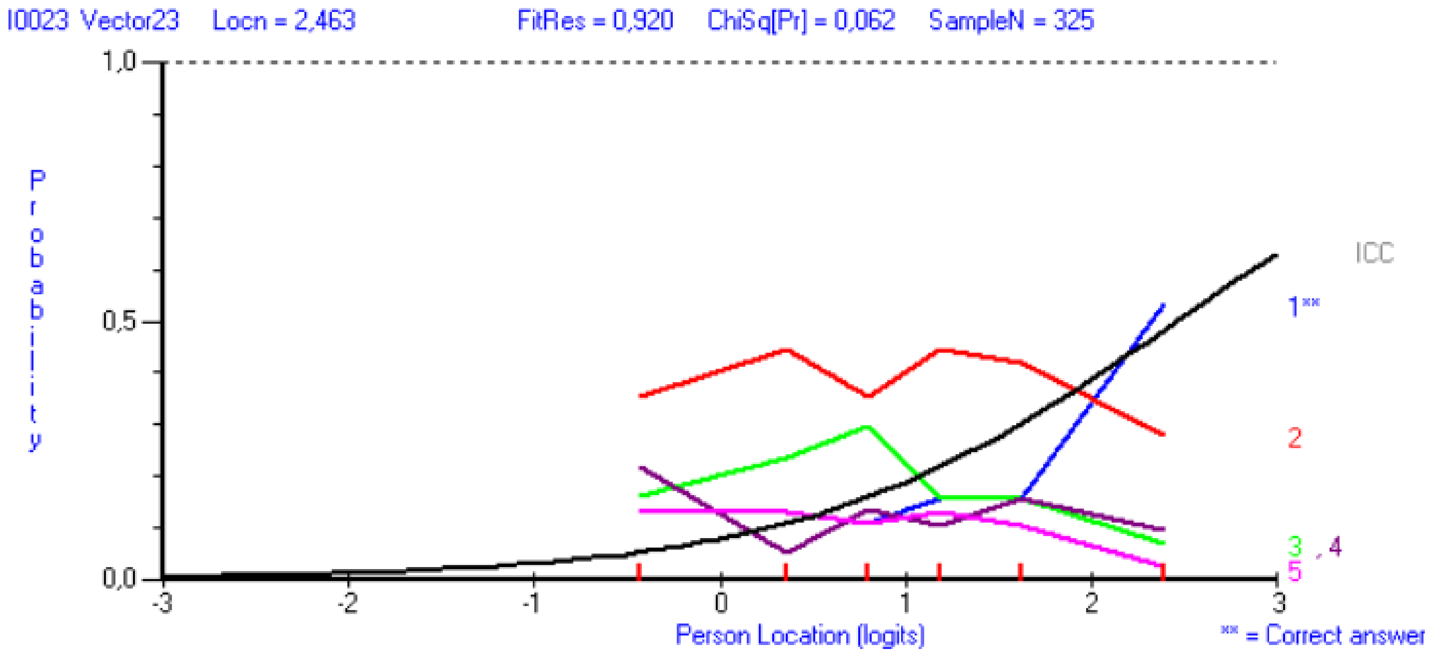
21. Distance from point to plane.

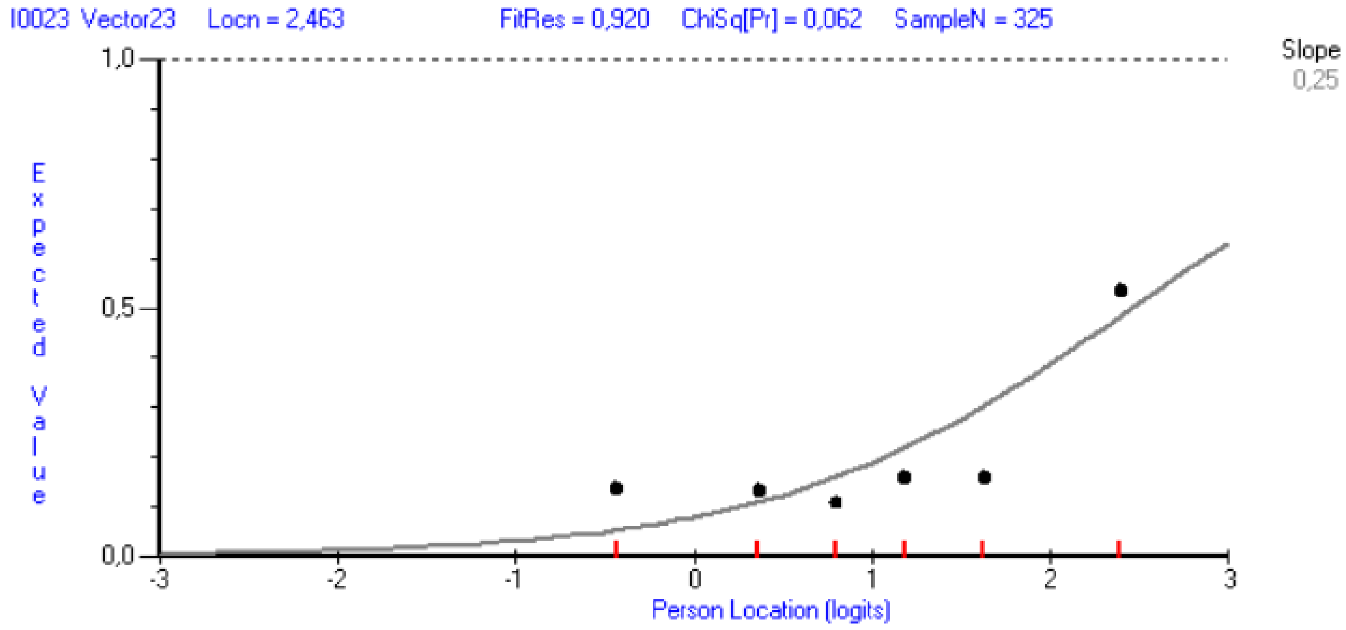
Item 14 curves





Item 23 curves





Typical errors

- Errors identified in the students' rough work included
  - Using the vector product
  - Drawing unnecessary 3-d diagrams
  - Confusion in using scalar product (incorrect use of modulus, unnecessary angle calculations, trying to write a scalar as a vector)
  - Errors in basic arithmetic

Apparent underpinning issue

The geometric role of the scalar product is not understood.

How the result of a scalar product can be interpreted is not known.

Situations in which scalar product is useful are not recognized.

Recommendations

- That both students and teachers not allow the simplicity of the scalar product notation to obscure the complexity of the contextual role of the scalar product.
- That many exercises (in lectures, tutorials, workshops, homework) be designed to approach the scalar product from a geometric or contextual point of view

References

- Craig, T.S., Cloete, T.J. (2015). Simple rule, hidden meaning: the scalar product in engineering mathematics. *Proceedings of the Tenth Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics (Elephant Delta), Port Elizabeth, 22-27 November 2015*,18-42.
- Craig, T.S. (2017). Challenging assumptions of notational transparency: the case of vectors in engineering mathematics. *International Journal of Mathematical Education in Science and Technology, 48*:sup1, S50-S66, DOI: 10.1080/0020739X.2017.1356390.
- Barniol, P., & Zavala, G. (2016). A tutorial worksheet to help students develop the ability to interpret the dot product as a projection. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(9), 2387-2398.