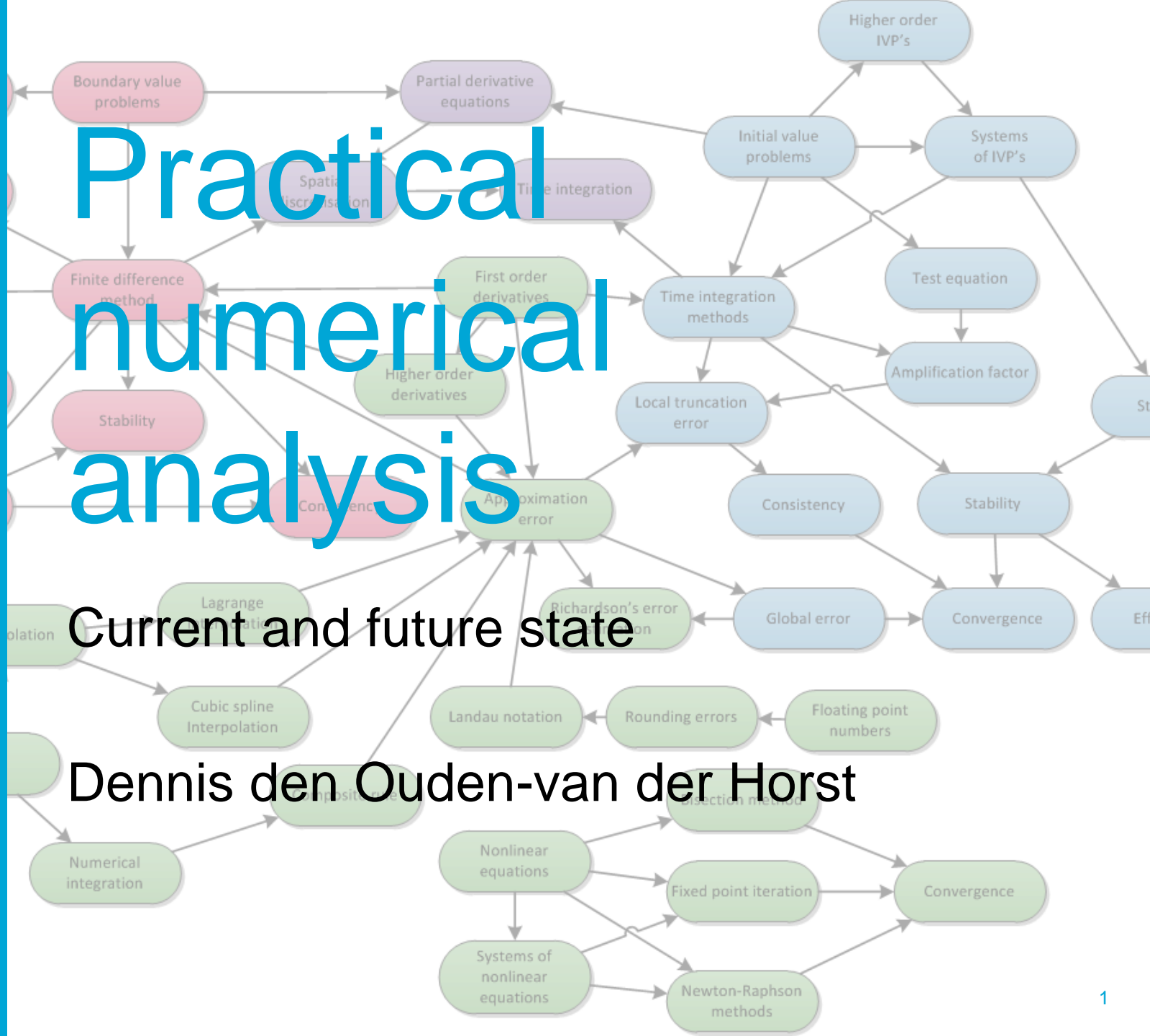


# Practical numerical analysis

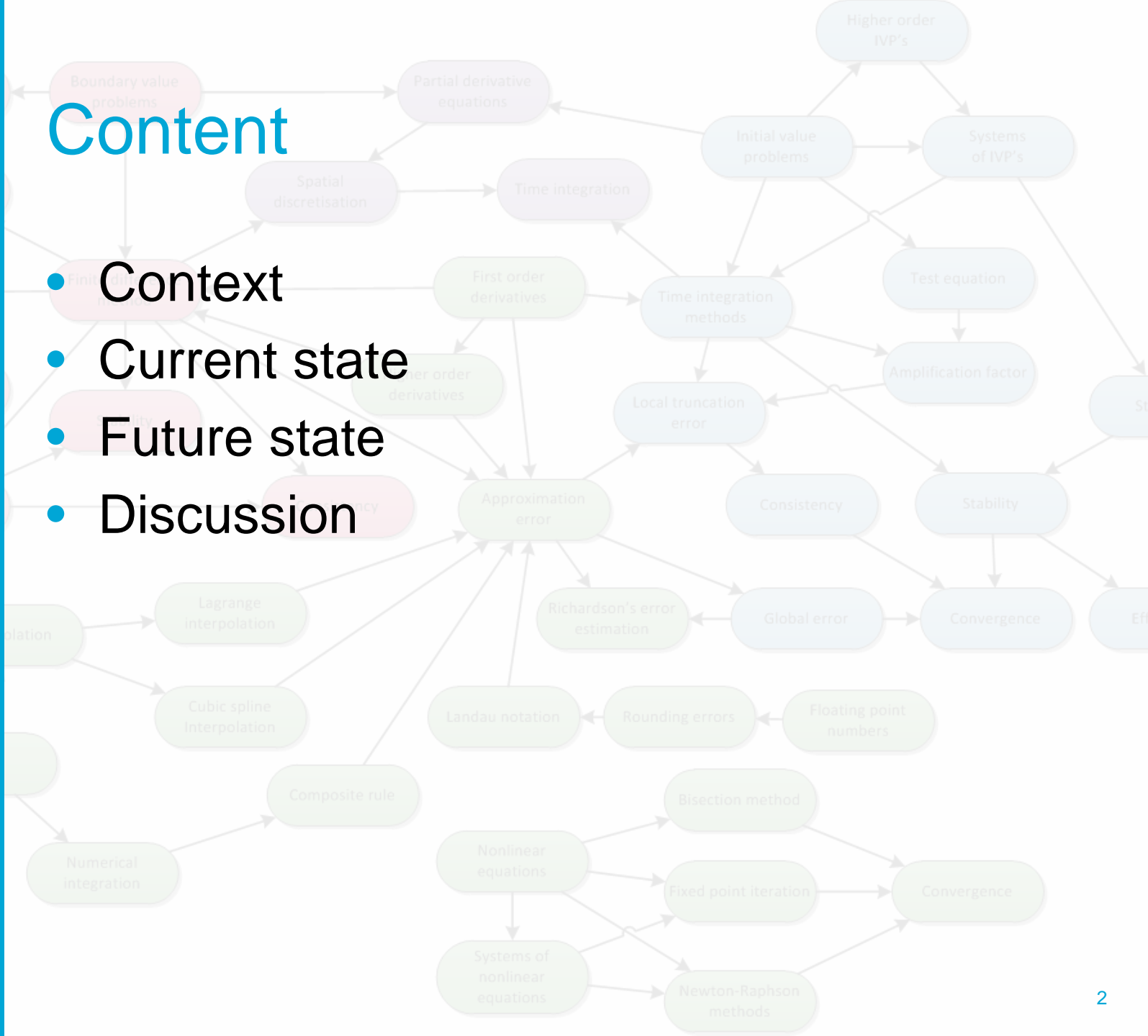
Current and future state

Dennis den Ouden-van der Horst

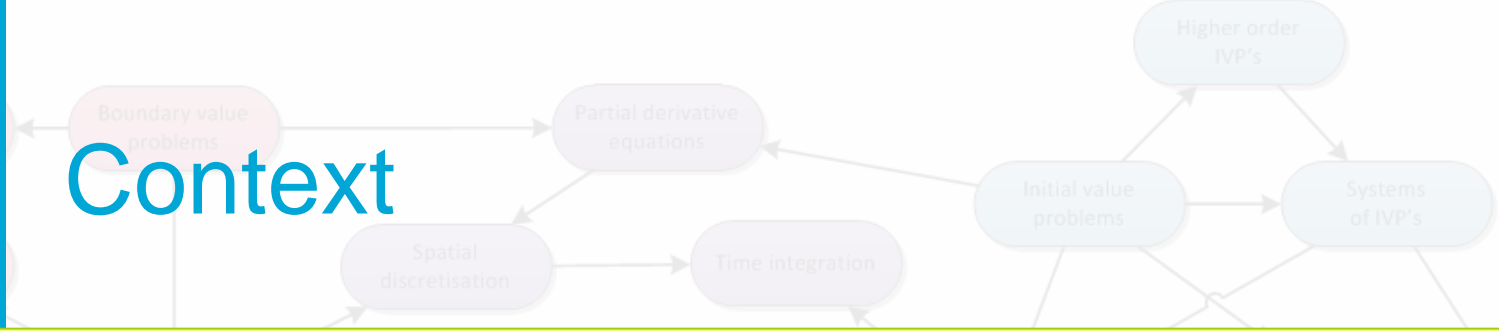


# Content

- Context
- Current state
- Future state
- Discussion



# Context

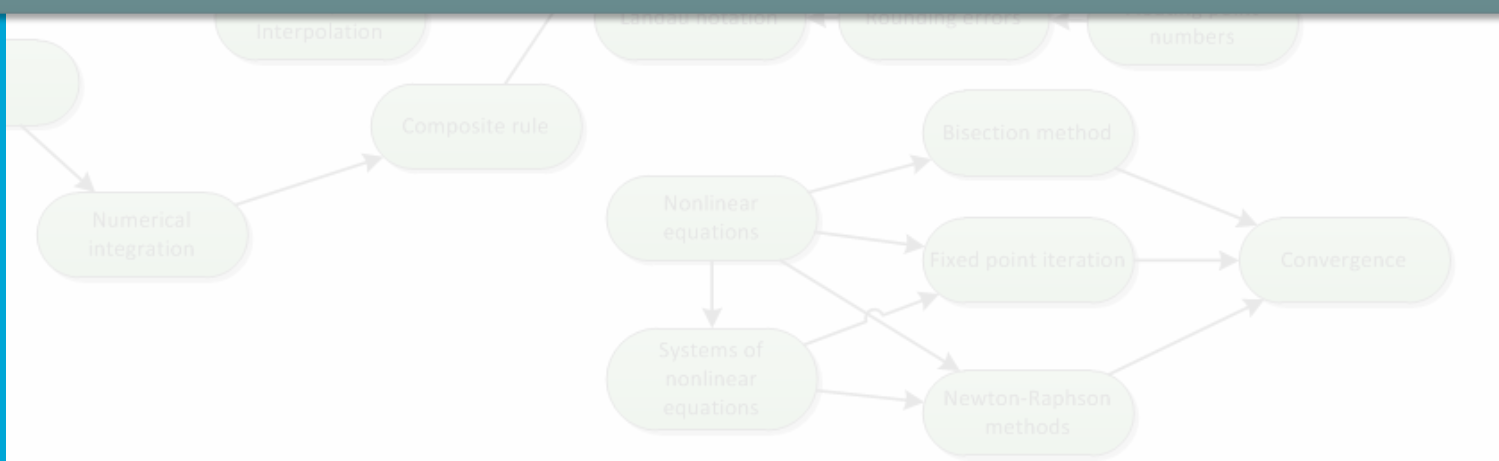


Q2

Earth Sciences Bachelor	100
Bridging minor Applied Mathematics	20
Free elective	20

Q4

Mechanical Engineering Bachelor	450
Maritime Engineering Bachelor	50
Bridging Programme 3mE	80



**Hint:** Let  $Q$  denote the amplification factor of Modified Euler. Numerical stability is guaranteed if

$$|Q(h\lambda_j)| \leq 1$$

Here,  $\lambda_j$  refer

8. Let us denote

$$\Lambda_h = \{h\lambda_j\}$$

Make a graph with the real axis must be  $x$  and  $y$  for the imaginary part of these points or

9. Transport (draw) the stability region of the resulting points answer to question 8.

**Hint:** With  $z = h\lambda_j$ ,  $|Q(z)| \leq 1$ . The complex point

**Remark:** Students should consult the Bode plot example can be found in the appendix.

10. Next, we investigate

$$h \leq 1.0$$

leads to a stability region. See the appendix (Figure 15.1).

It will be clear that

11. Eigenvalues of

$$e^{i\omega t} = \cos(\omega t)$$

in the solution will rise to a complex number. The contribution

term will introduce a period  $T = 2\pi/\Omega$  in the problem, with  $\Omega$  in the range presented in (5).

Give an upper bound for the time step  $h$ , if at least 10 time steps per smallest period need to be taken.

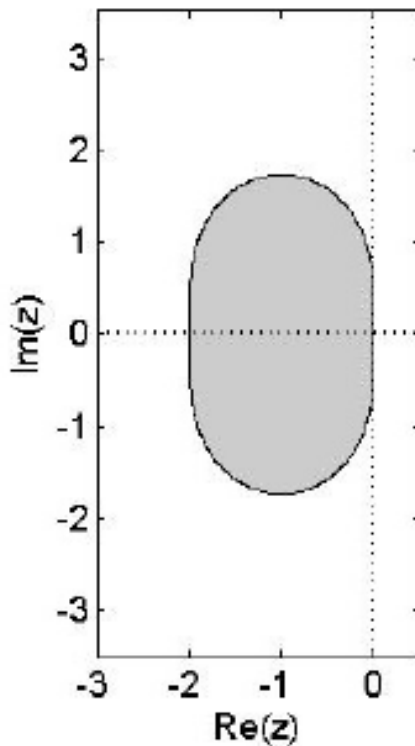
the scaled external force  $u/k_1$  in one figure for this value of  $\Omega$ . It seems that the two forces that are working on mass  $m_1$  cancel each other out for this value of  $\Omega$ . Can you express this  $\Omega$  in terms of  $m_2$ ,  $k_1$  and  $k_2$ ? Does this cancellation of forces occur for every initial value?

method? Present your answer and confine yourself to

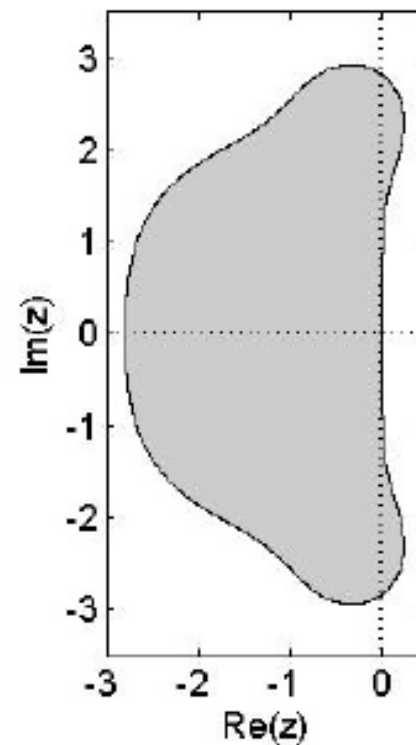
fixed time  $t$  (say  $t = 2h$ ). Let us denote the error  $E$  in

## Appendix: Stability regions of Modified Euler and RK4.

Stability region, Modified Euler



Stability region, RK4



method.

answers the question of the Matlab

time interval. The error is small.

interval  $[0, 40]$ . The system will

solutions computed. The error is small.

interval  $[0, 40]$ . In question 15. In

working on mass

the time interval  $[0, 40]$ . For which value of  $\Omega$  is the maximum almost equal to zero? Plot the positions and

## Problem description

A spring-mass system consists of two masses,  $m_1$  and  $m_2$ , which can only move horizontally (see the picture below). The masses are connected by means of a spring with spring constant  $k_1$ . The left mass is also connected to the wall by means of a spring with spring constant  $k_2$ . A periodic force  $u$  is working on the right mass with amplitude 3 Newton and frequency  $\Omega$ . The positions (deviations from rest) of the masses  $m_1$  and  $m_2$  will be denoted by  $x_1$  and  $x_2$  respectively. The influence of gravity will be neglected. We are

### Assignment 3

To have an accurate estimate for  $\Omega$ , the errors in your solutions should not be too large. To determine an appropriate time step  $\Delta t$ , answer the following questions:

- Find a formula based on Richardson's extrapolation which estimates the error in the numerical solution at  $t = 1$  using only numerical solutions.
- Use this formula to find a value for the time step  $\Delta t$  such that the absolute errors in  $x_1$  and  $x_2$  are less than  $10^{-5}$ . Use  $\Omega = 5$  in your calculations<sup>4</sup>. Make sure  $1/\Delta t$  is an integer.

### Assignment 4

Using the value for  $\Delta t$  you found in the previous assignments, answer the research question. That is, find the answer to the question:

*What is the lowest value of  $\Omega > 0$  such that  $|x_1(t)|$  for  $t \in [0, 40]$  is as low as possible?*

10.0	-1.3114e-01	-9.8071e-02
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Hint: To be able to choose an  $\epsilon$  for a system of second-order differential equations to a system of first-order differential equations.

To be able to perform this practical assignment, knowledge and understanding of Chapters 3 and 6 of the book are required. After performing the assignments, you must write a report.

# Discussion

**Feedback and questions on the future state are welcomed.**