SHIPS, WAVES AND MATH

Mathematics & Water, Deltares, 13 November 2014

MARIN, Ed van Daalen

CONTENTS

- MARIN
- Application of math to ship hydromechanics
- Conclusions

MARITIME RESEARCH INSTITUTE NETHERLANDS


Wageningen


Ede


Houston

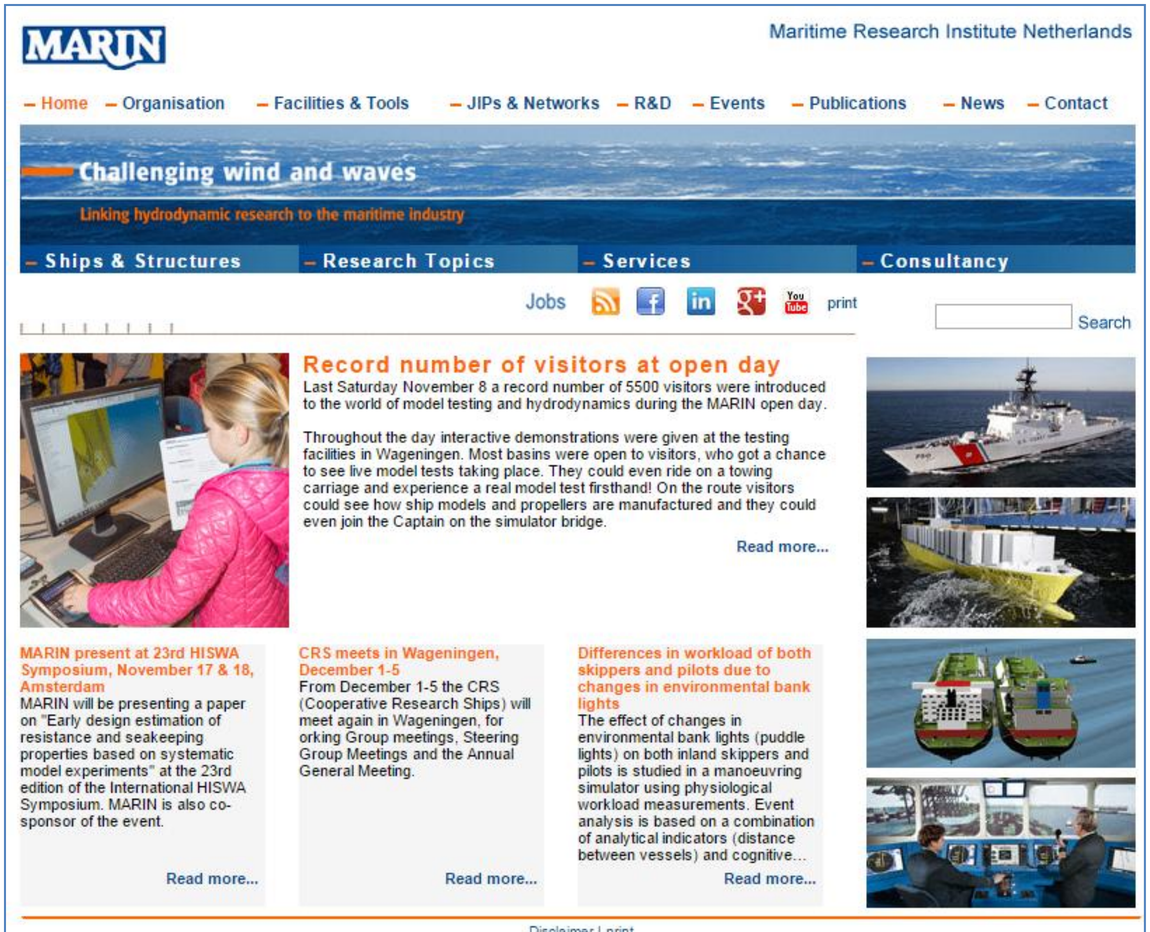
- hydrodynamic research for maritime industry, nonprofit
- founded 1929, 7 model basins, 350 employees, 42 M€ turnover
- model tests, trials & full scale monitoring, simulations
- international market: design companies, shipyards, classification, ship operators

MARIN ORGANISATION

- Ships: powering & resistance, seakeeping, manoeuvring for all ship types
- Offshore: on/offloading, drilling platforms, windmill installation
- Nautical Simulator: harbour design, training
- Trials and Monitoring: full scale measurements
- Software: simulation
- Production: model factory, instrumentation
- Research and Development: fundamental developments in experiments and simulations

LEARN MORE ABOUT MARIN

- www.marin.nl (nice company video!)
- www.youtube.com/marinmultimedia

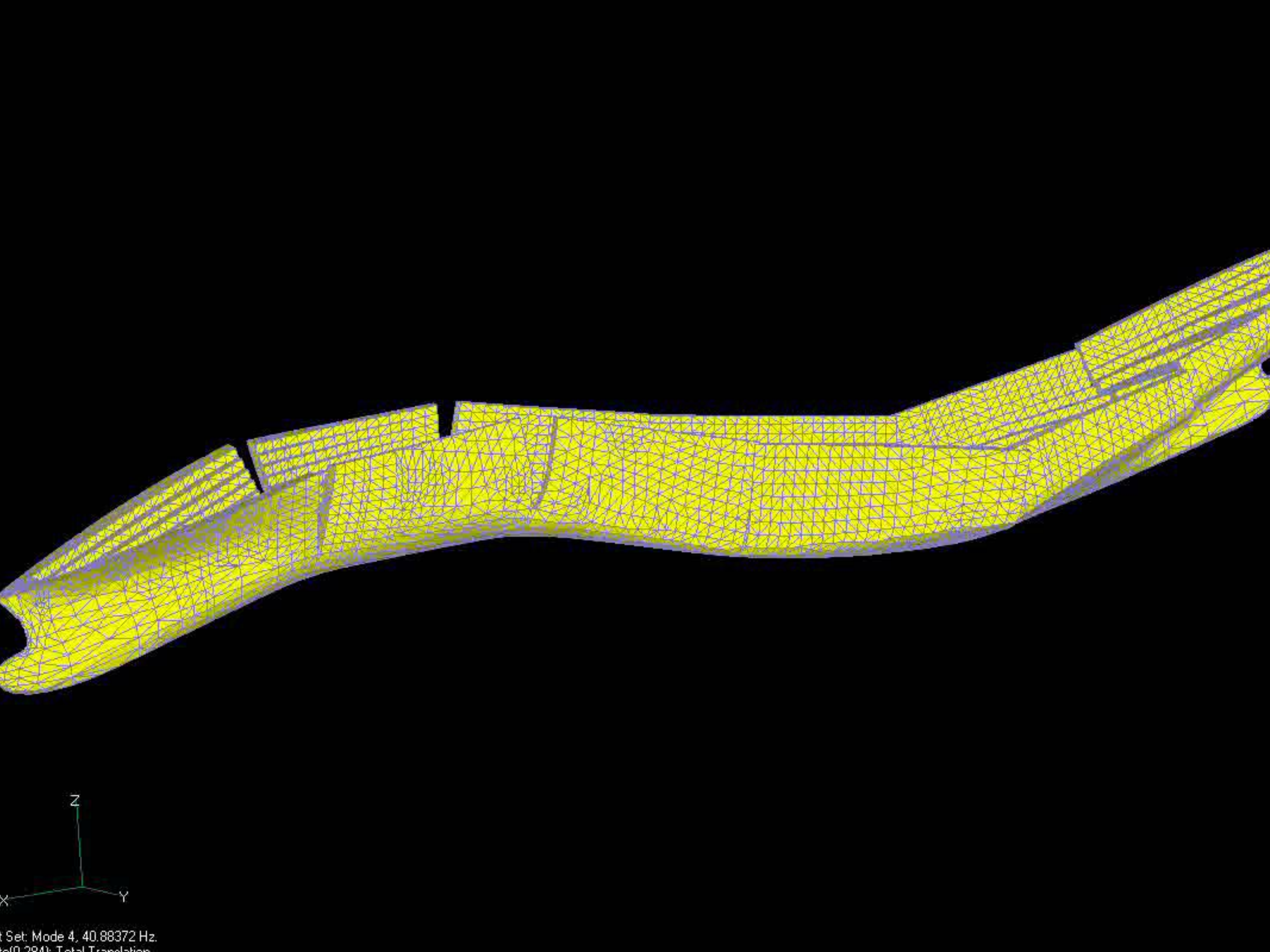


Life can be beautiful …

... BUT SOMETIMES LIFE IS HORRIBLE ...

- Herald of Free Enterprise
- Estonia
- Costa Concordia
- ...

How can we help to avoid this?


Z
X
Y
t Set: Mode 4, 40.88372 Hz.

LNG carriers

- **Sloshing in liquid cargo tanks**

How can we help ?

heavy cargo

- structural
- (off)loading

How can we help ?

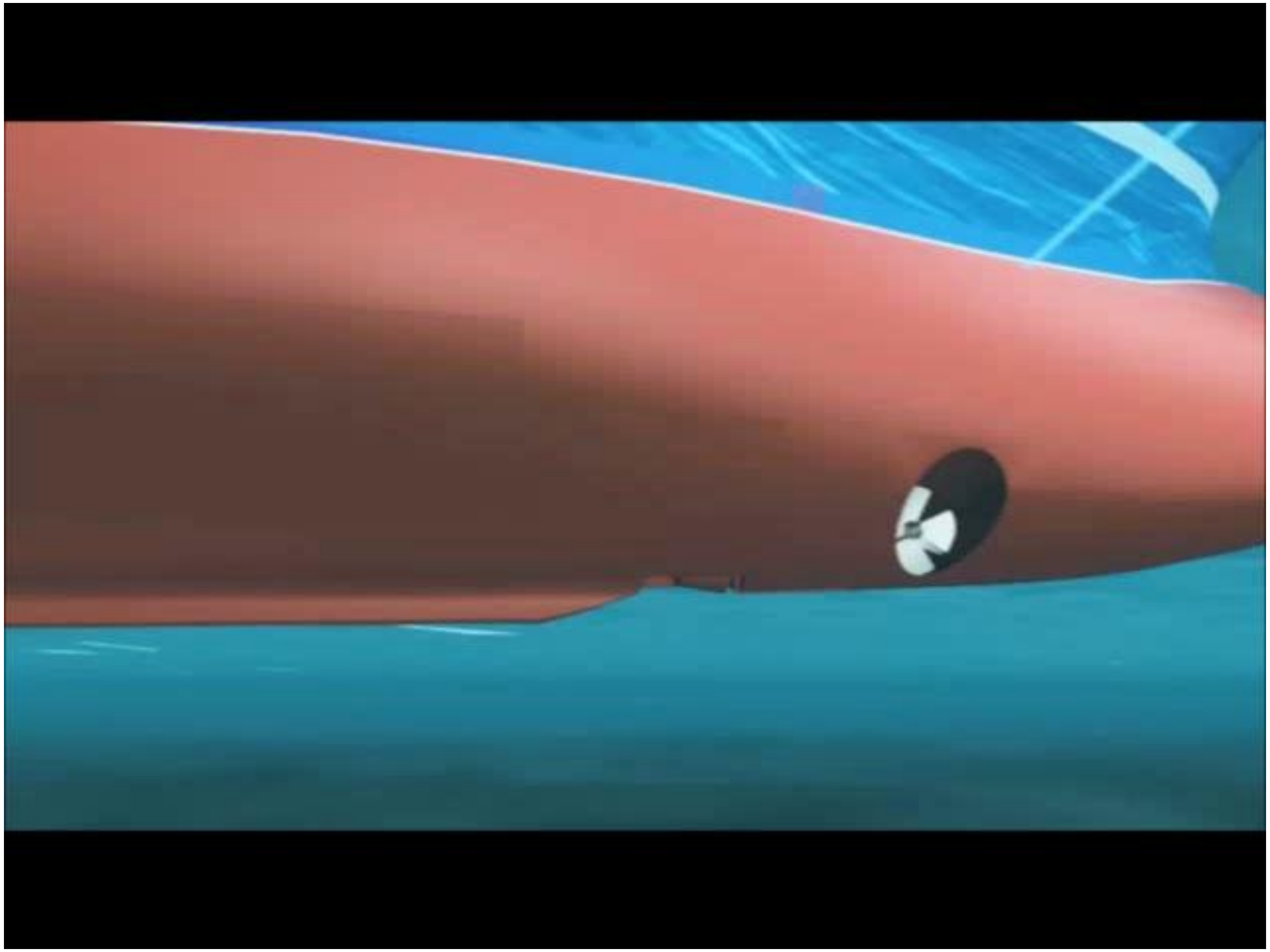

bad weather
• high waves
• high loads
How can we help ?
PARTY
FLOCK

bad weather

- **comfort**
- **operability**
- **safety**

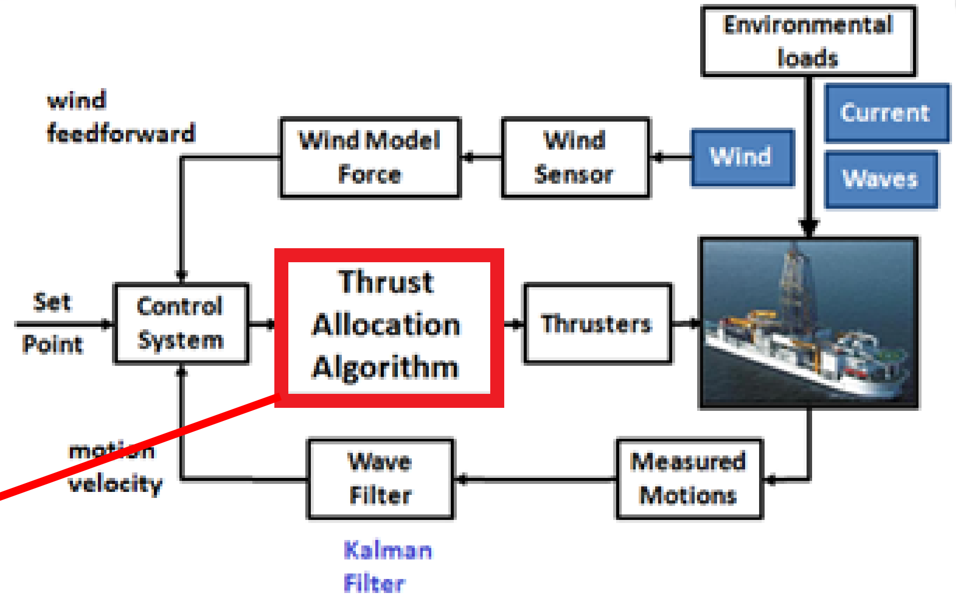
How can we help ?

THRUST ALLOCATION



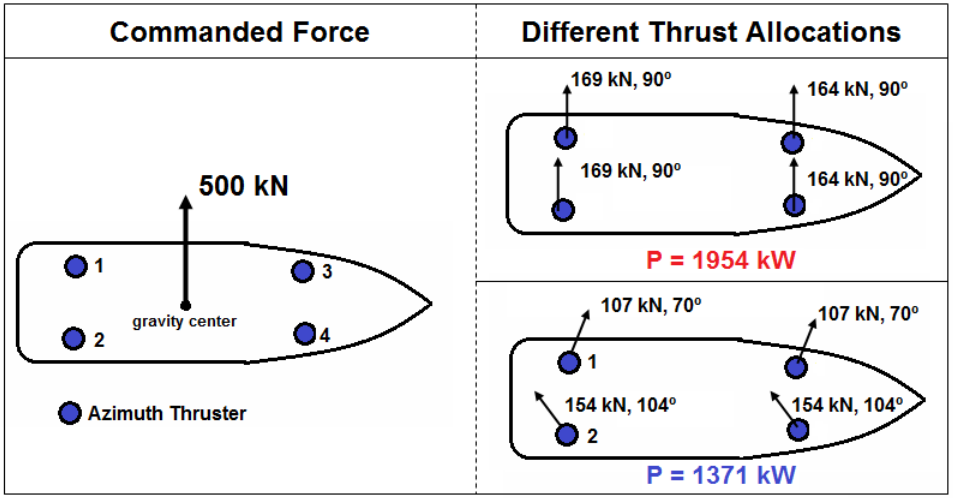
THRUST ALLOCATION - OBJECTIVES

Dynamic Positioning (DP) System

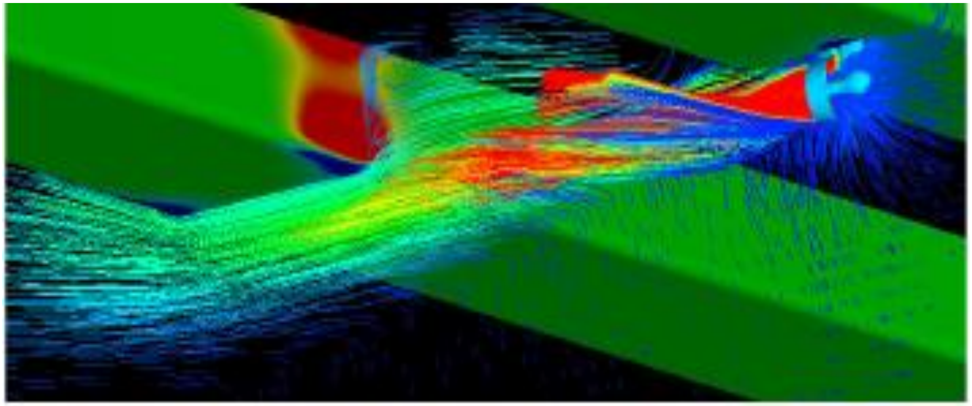


Thrust Allocation Algorithm

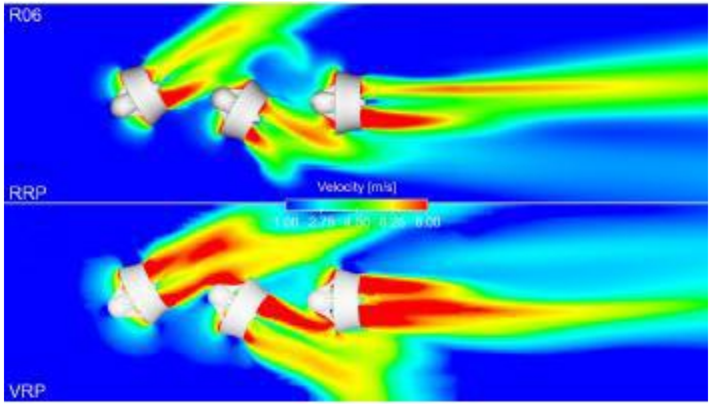
- match required forces
- minimize power
- account for hydrodynamic interaction effects
- respect physical limitations
  - maximum rpm change
  - maximum azimuth change



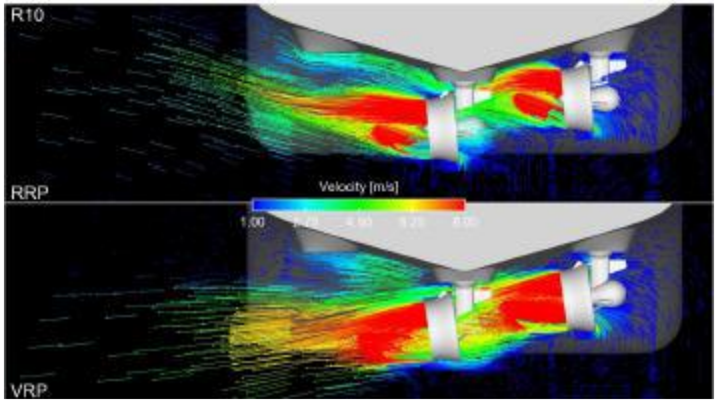
THRUST ALLOCATION – INTERACTION EFFECTS



thruster-hull interaction



thruster-current interaction

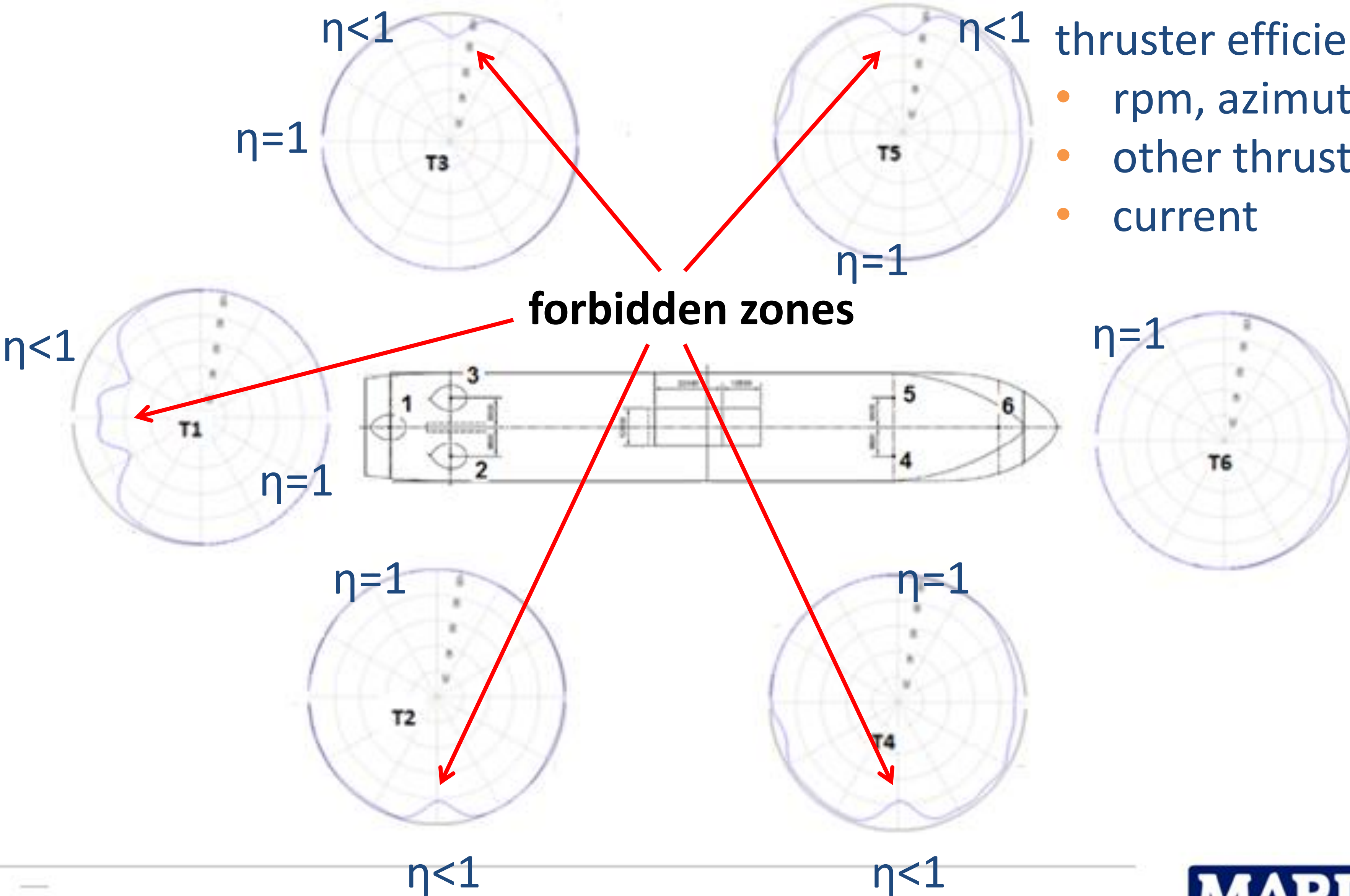


thruster-thruster interaction

THRUST ALLOCATION - EFFICIENCY FUNCTIONS

thruster efficiency

- rpm, azimuth
- other thrusters
- current



THRUST ALLOCATION - OPTIMIZATION PROBLEM

$$\underset{T,\alpha}{Min} \sum_{i=1}^{N} c_i(\rho, D) \cdot \frac{C_{Q_i}}{C_{Q0_i}} \cdot T_i^{\frac{3}{2}}$$

power minimization

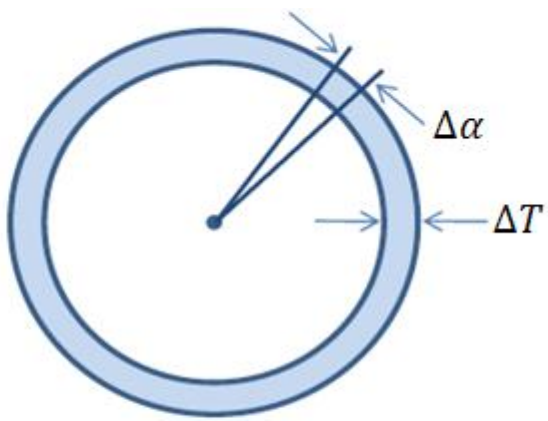
$$R: \begin{cases} \sum_{i=1}^{N} T_i \cdot \cos\alpha_i \cdot \eta_i - F_x = 0 \\ \sum_{i=1}^{N} T_i \cdot \sin\alpha_i \cdot \eta_i - F_y = 0 \\ \sum_{i=1}^{N} [T_i \cdot \eta_i \cdot (x_i \cdot \sin\alpha_i - y_i \cdot \cos\alpha_i)] - M_z = 0 \end{cases}$$

generate required forces

⇩

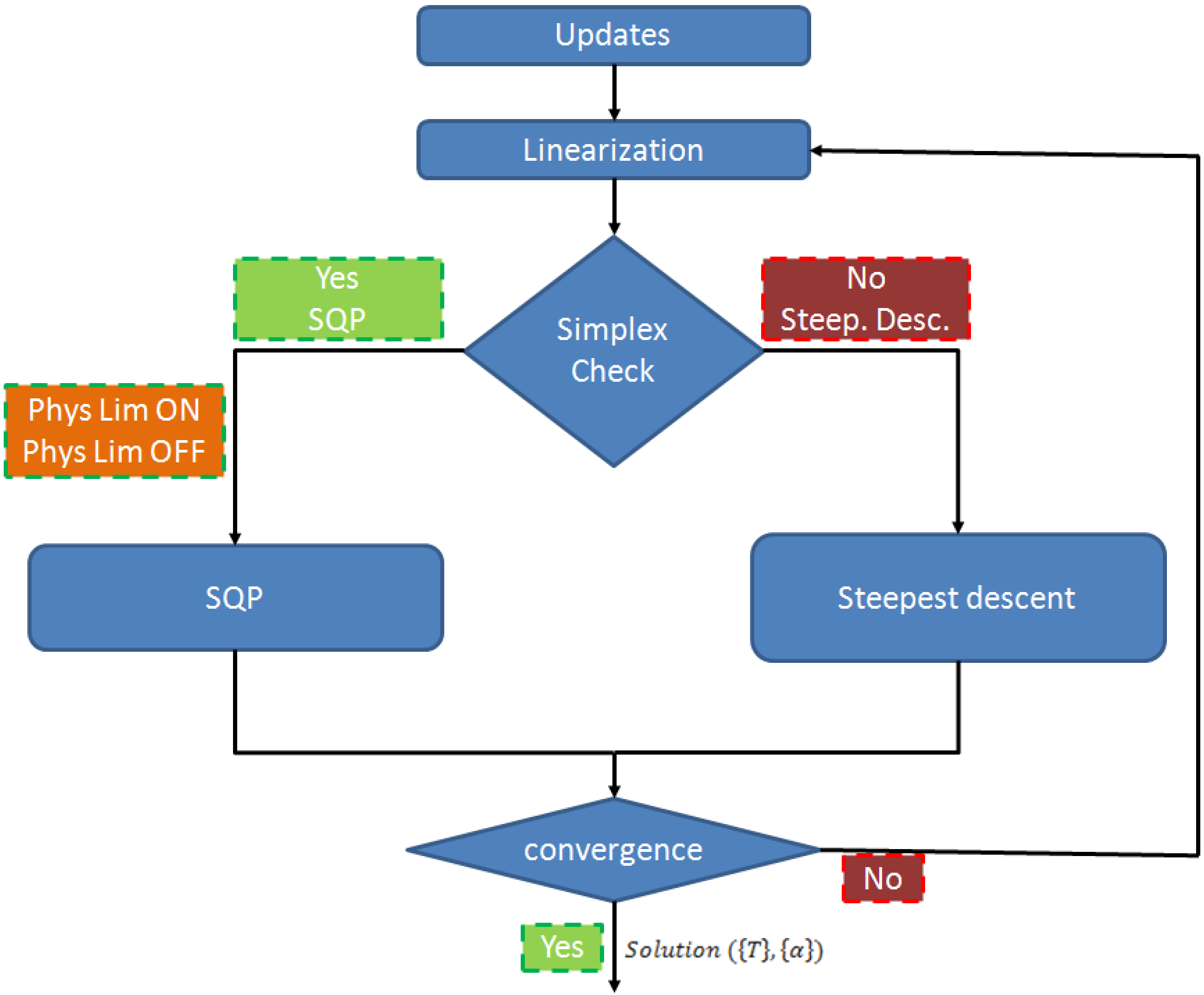
account for hydrodynamic interactions

$$I: \begin{cases} T \leq T_{max} \\ \Delta\alpha \leq \Delta\alpha_{max} \\ Lb \leq T \leq Ub \end{cases}$$



physical limitations

THRUST ALLOCATION - ALGORITHM



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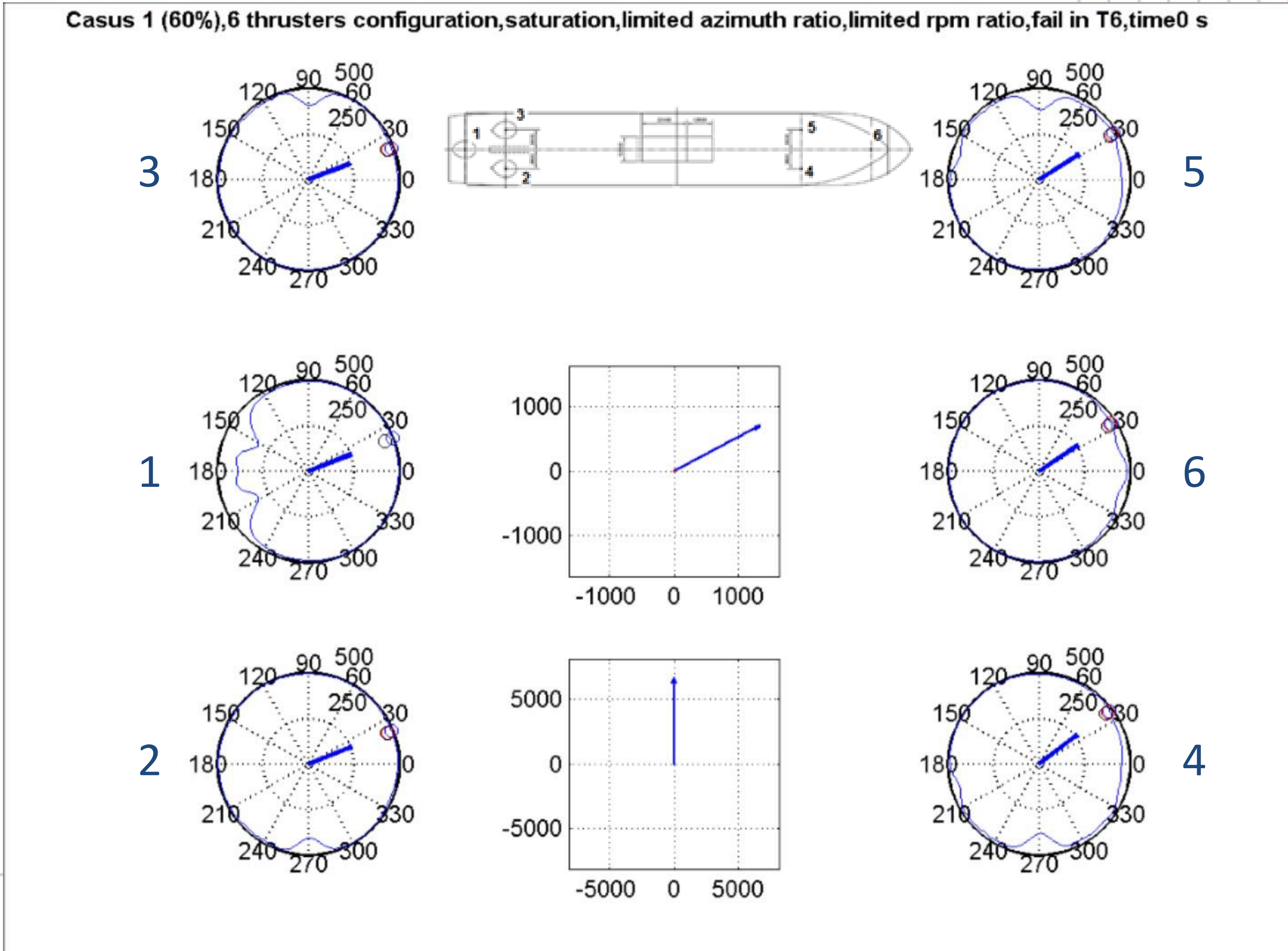
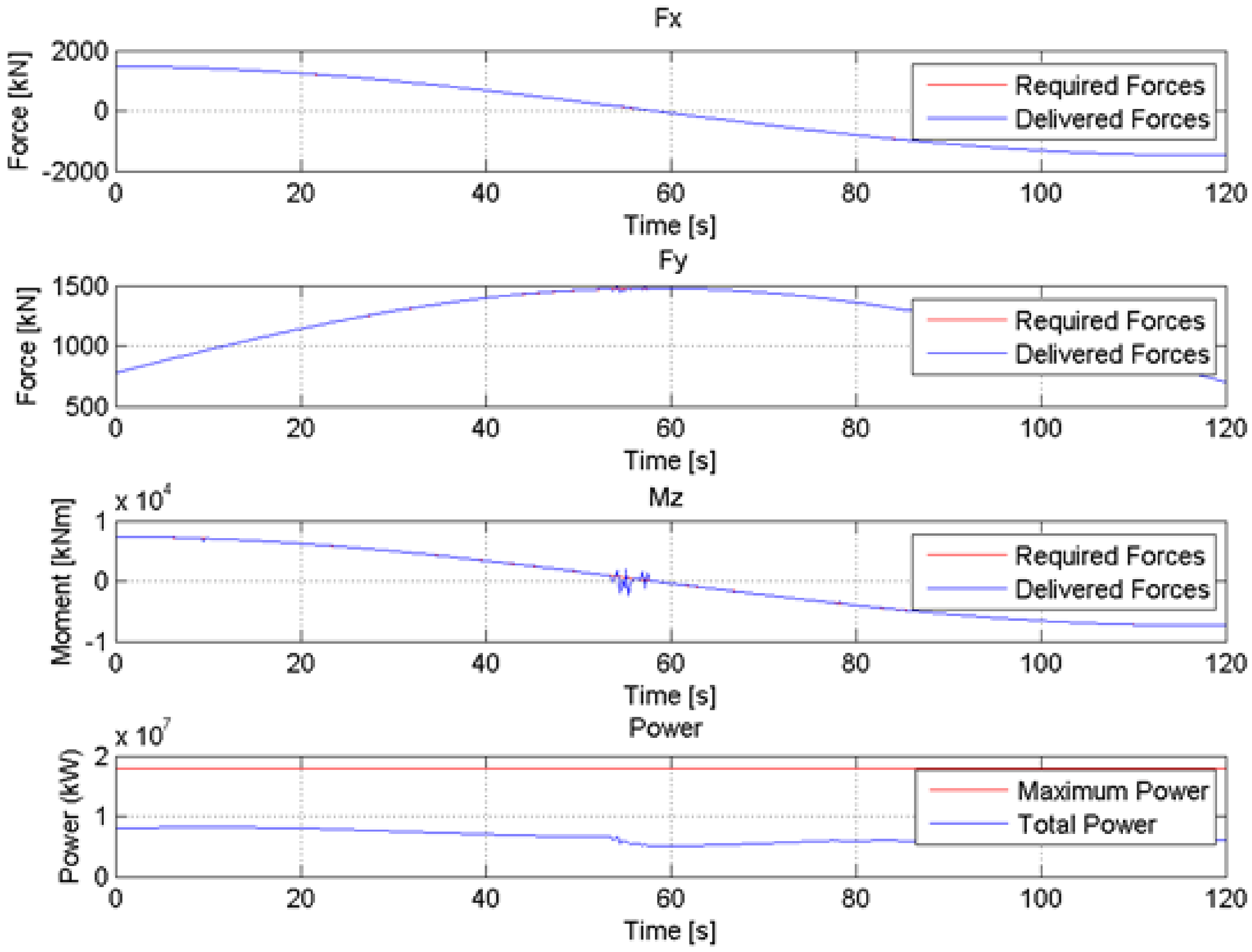
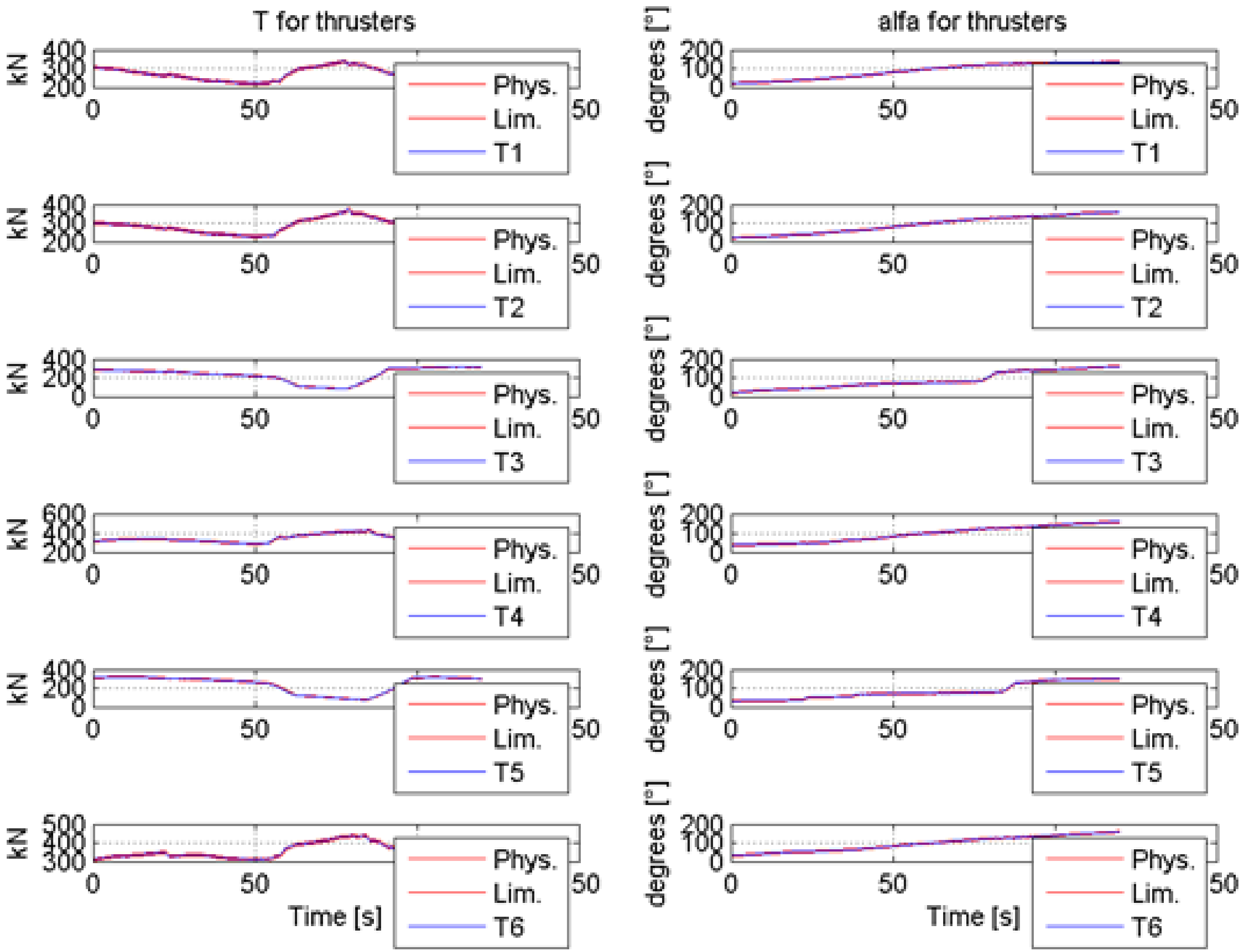
THRUST ALLOCATION - CROSSING FORBIDDEN ZONES



THRUST ALLOCATION – MATCH REQUIREMENTS



THRUST ALLOCATION – RESPECT PHYSICAL LIMITS



MANOEUVERING EQUATIONS

research started at SWI 2011

MANOEUVERING EQUATIONS – MATHEMATICAL MODEL

- maneuvering model: set of <u>coupled</u> ordinary differential equations (ODEs) describing *ship motions in calm water*, including <u>nonlinear hull forces</u> and <u>nonlinear propulsion forces</u>

- many hull parameters (~ 30) and propulsion parameters (~ 20) involved

- many of these parameters are determined by experiments (scale models) and CFD

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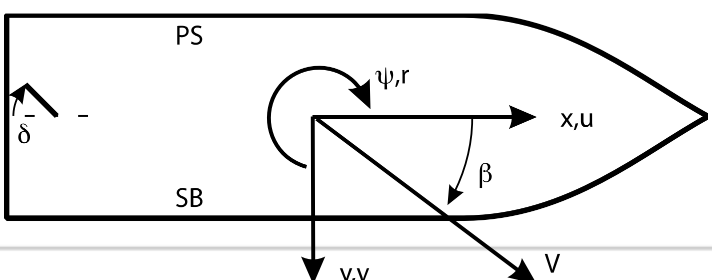
- Propeller-Rudder Model: used in MARIN maneuvering simulation program SurSim

$$
\begin{aligned}
L_{pp}\left((m'+m'_{uu})\dot{u}\right) &= m'r'v + X'_H + X'_R + X'_P \\
L_{pp}\left((m'+m'_{vv})\dot{v} + m'_{vr}\dot{r}'\right) &= -m'r'u + Y'_H + Y'_R \\
L_{pp}\left(m'_{rv}\dot{v} + (I'_{zz}+m'_{rr})\dot{r}'\right) &= + N'_H + N'_R
\end{aligned}
$$

- (simplified) Thruster Model:

$$
\begin{aligned}
L_{pp}\left((m'+m'_{uu})\dot{u}\right) &= m'r'v + X'_H + \tau\cos\alpha \\
L_{pp}\left((m'+m'_{vv})\dot{v} + m'_{vr}\dot{r}'\right) &= -m'r'u + Y'_H + \tau\sin\alpha \\
L_{pp}\left(m'_{rv}\dot{v} + (I'_{zz}+m'_{rr})\dot{r}'\right) &= + N'_H + x'_T\,\tau\sin\alpha
\end{aligned}
$$



$$\rightarrow \underline{\dot{u}} = \underline{F}(\underline{u}, \underline{\lambda})$$

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MANOEUVERING EQUATIONS - SOLUTIONS

$$\dot{\underline{u}} = \underline{F}(\underline{u}, \underline{\lambda})$$

obvious thing to do = direct simulation

→ time integration with initial conditions

- constant propulsion parameters: e.g. straight line, turning circle
  - NOTE for these motions $\dot{\underline{u}} = \underline{0}$
- time-dependent propulsion parameters: e.g. zig-zag manoeuver
  - NOTE for these periodic motions $\dot{\underline{u}} \neq \underline{0}$

Alternative: Numerical Continuation Method

NCM = a robust and fast method to

- find parameter-dependent set of ‘equilibria’ of ODEs

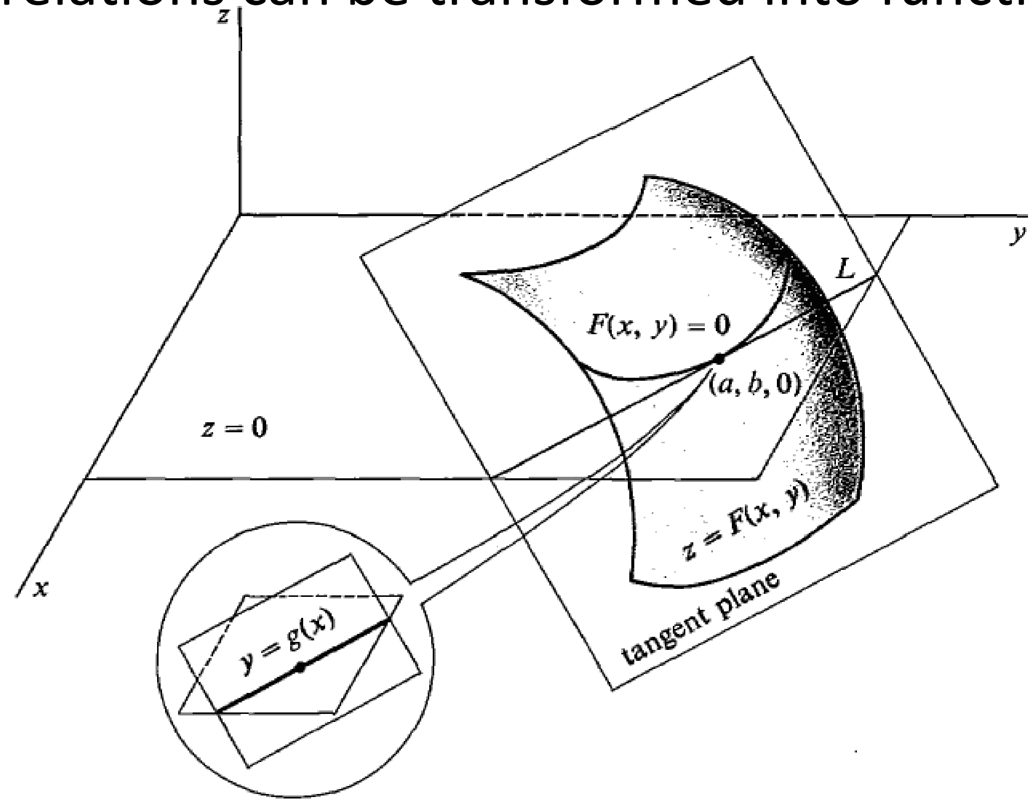
  (equilibrium = steady / stationary state solution)

- determine stability properties of equilibria

- find bifurcations and e.g. trace periodic solutions

  (bifurcation = transition from stable to unstable)

NCM is based on Implicit Function Theorem, stating that « relations can be transformed into functions »



$$\underline{\dot{u}} = \underline{F}(\underline{u}, \underline{\lambda}) \rightarrow \underline{0} = \underline{F}(\underline{u}, \underline{\lambda})$$

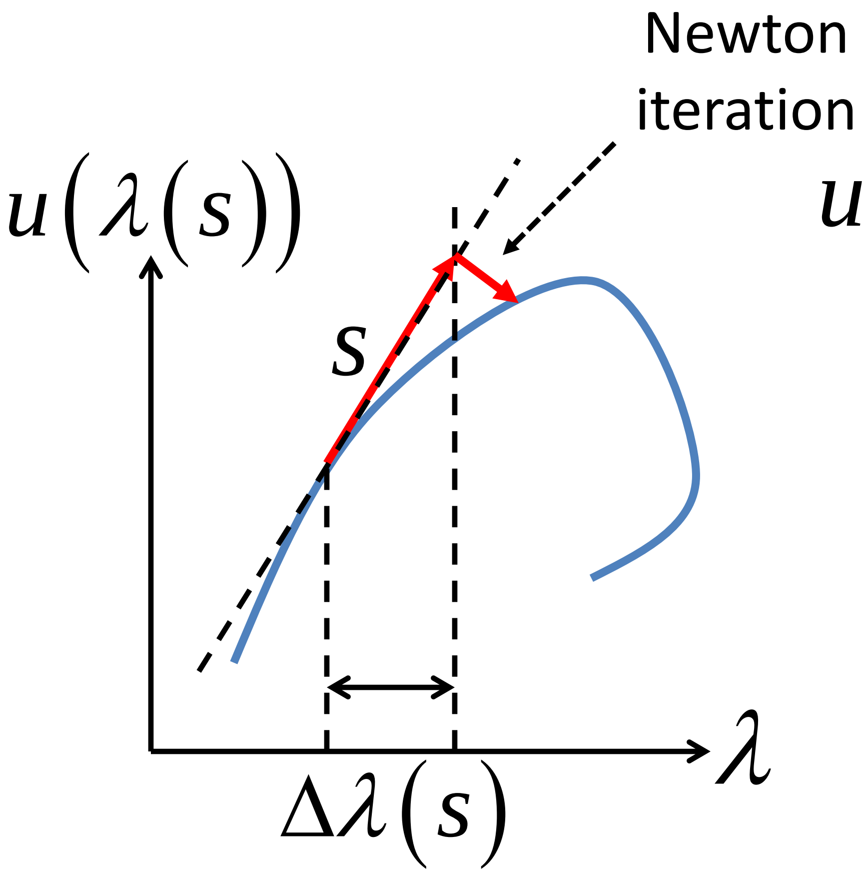
u: **n-vector (state variables, n=3, 4)**

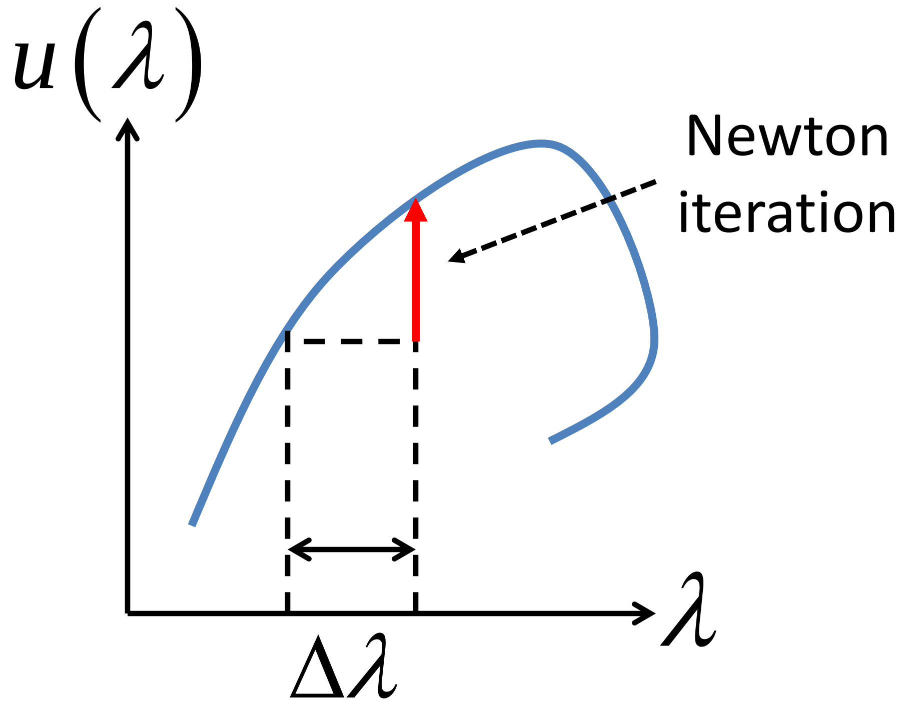
λ: **continuation parameters (select 1)**

MANOEUVERING EQUATIONS - NCM WITH AUTO

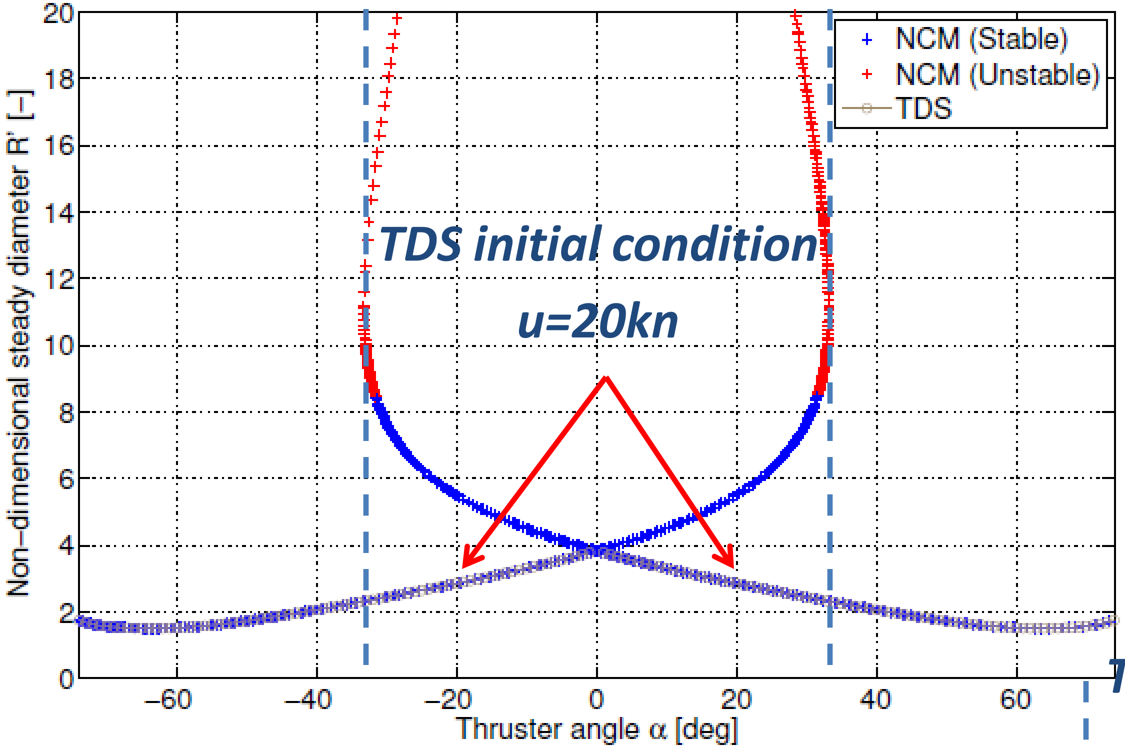
pseudo arc-length continuation (AUTO)

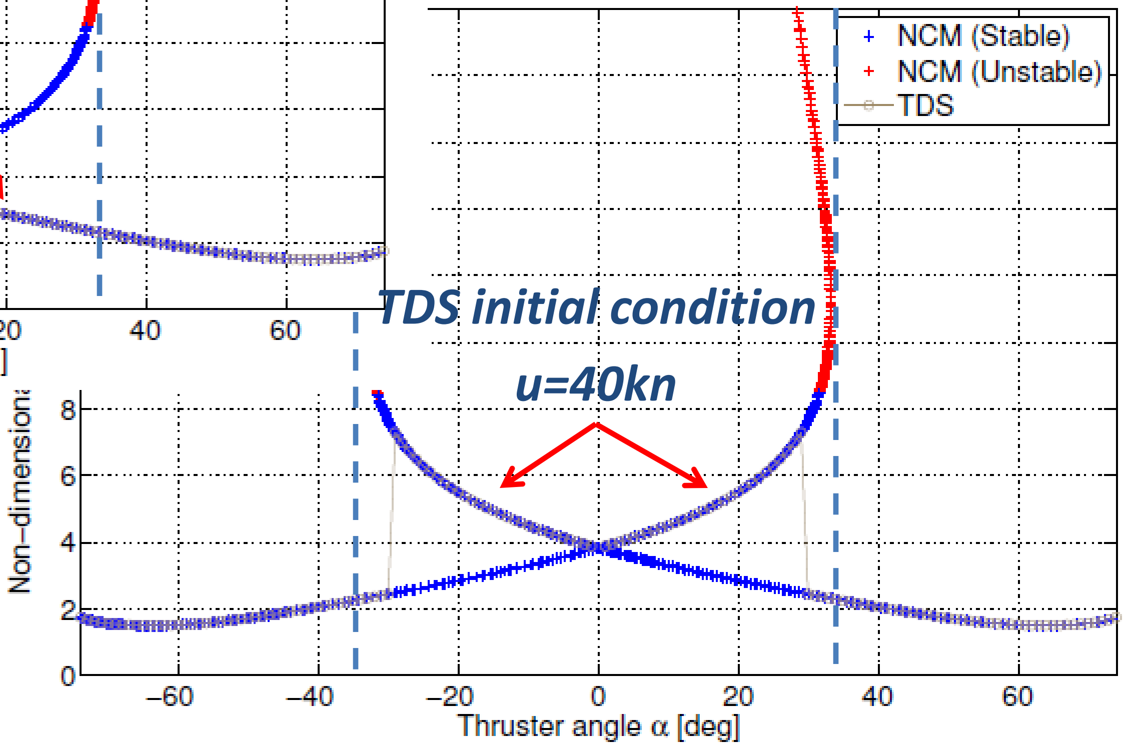
natural parameter continuation





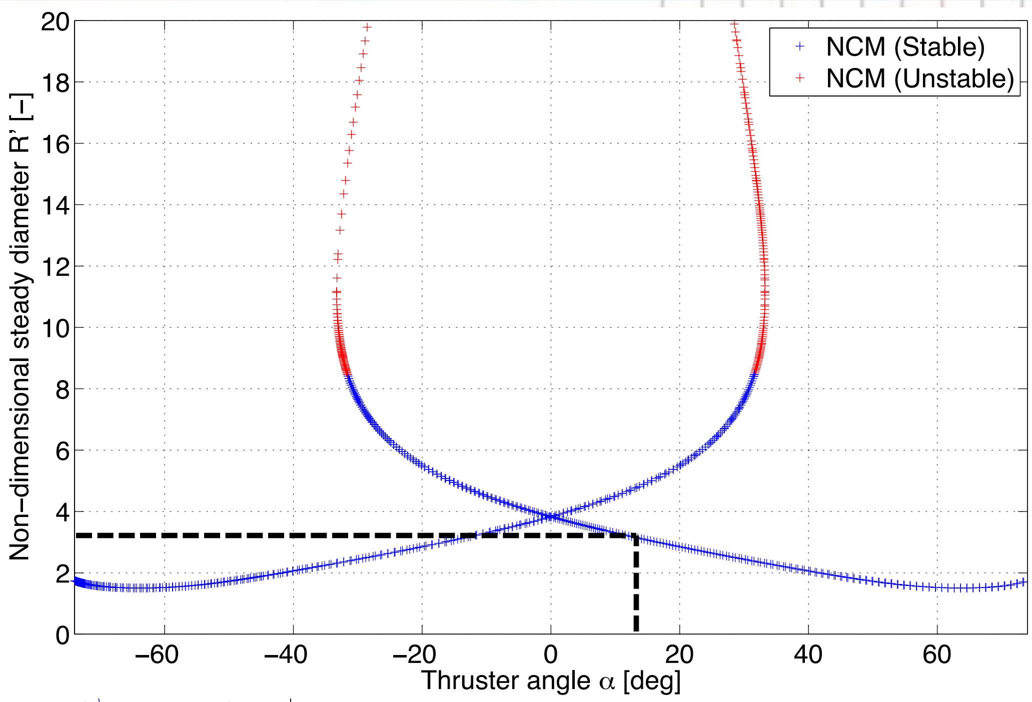
MANOEUVERING EQUATIONS - TURNING CIRCLE

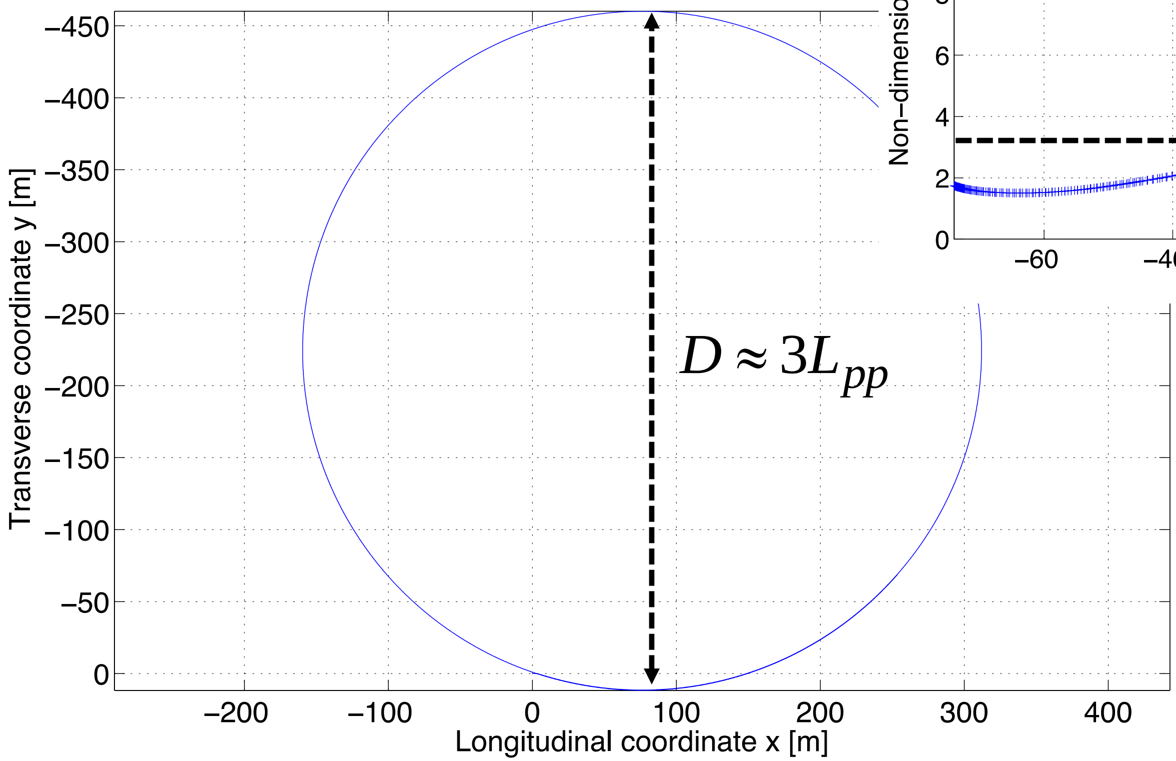




influence of the thruster angle

MANOEUVERING EQUATIONS - TURNING CIRCLE

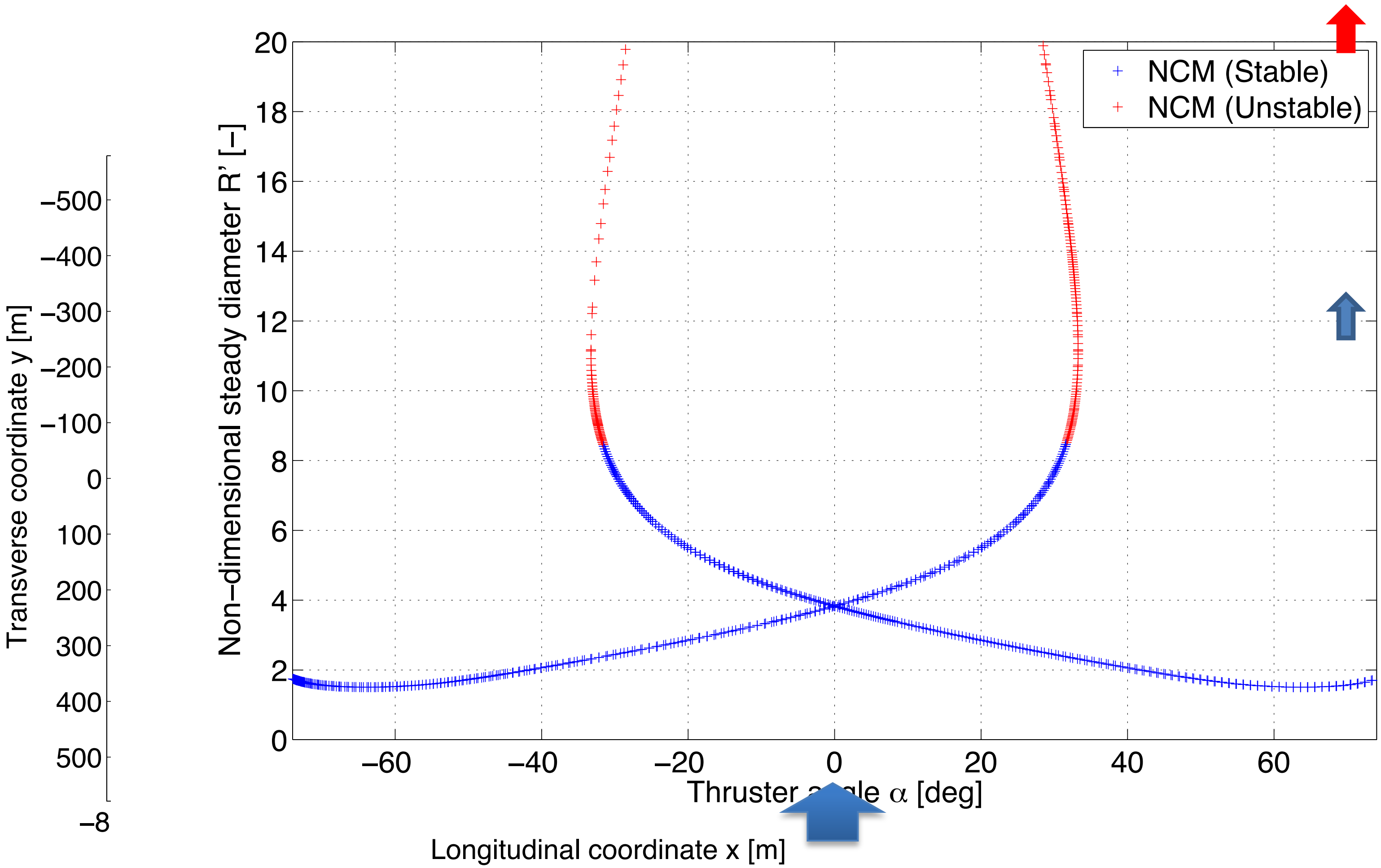




α=15deg

circular motion with noise (stable)

MANOEUVERING EQUATIONS - STRAIGHT LINE



straight line motion with noise (unstable)

MANOEUVERING EQUATIONS - YAW CONTROL

add yaw as state variable

$$\underline{u} = (u, v, r', \psi)$$

add extra ODE:

$$\dot{\psi} = r$$

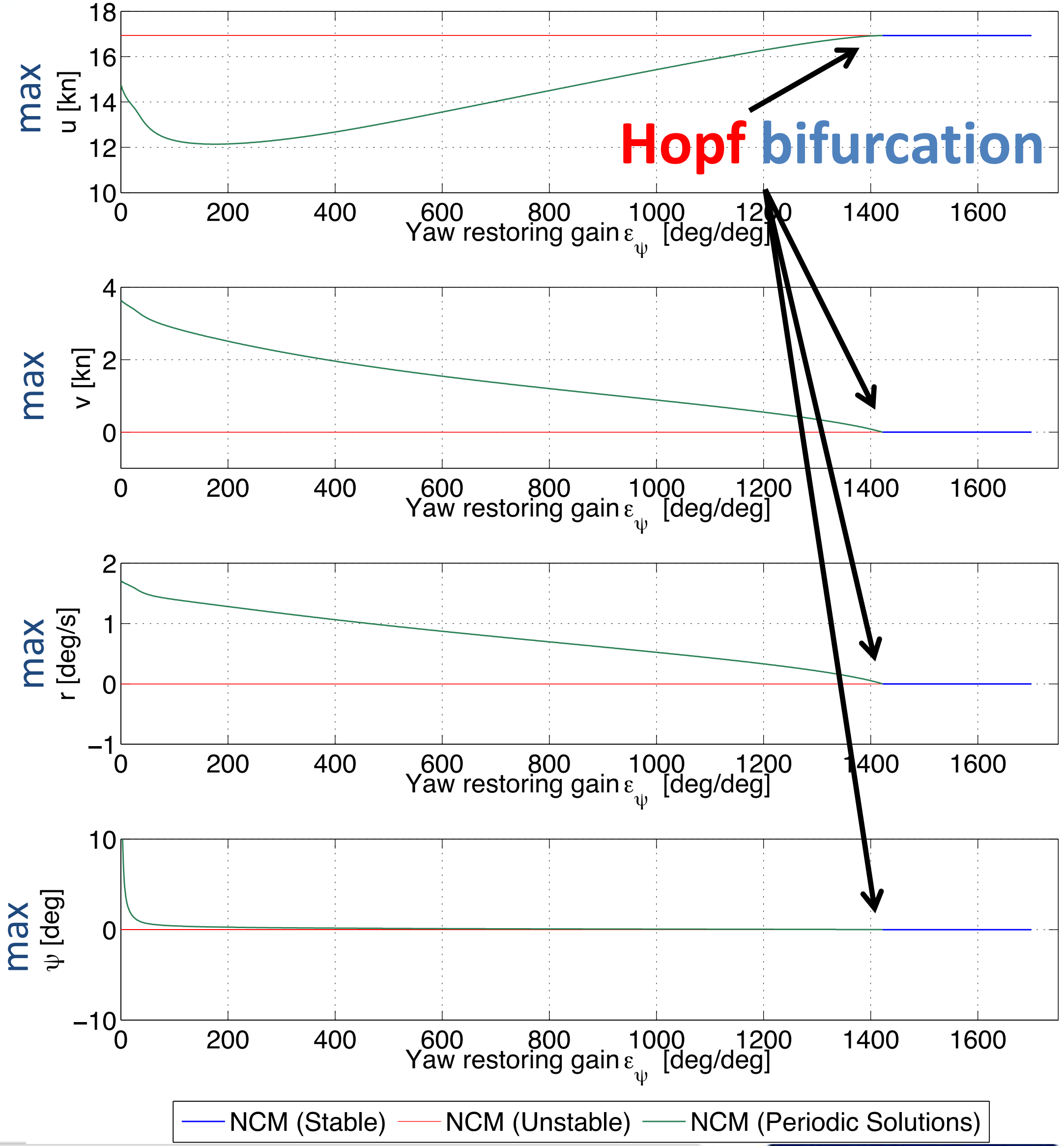
add yaw restoring control:

$$\alpha = \alpha_0 + \varepsilon_\psi (\psi - \psi_0)$$

$$\alpha_0 = 0$$

(α is a parameter,

not a state variable!)



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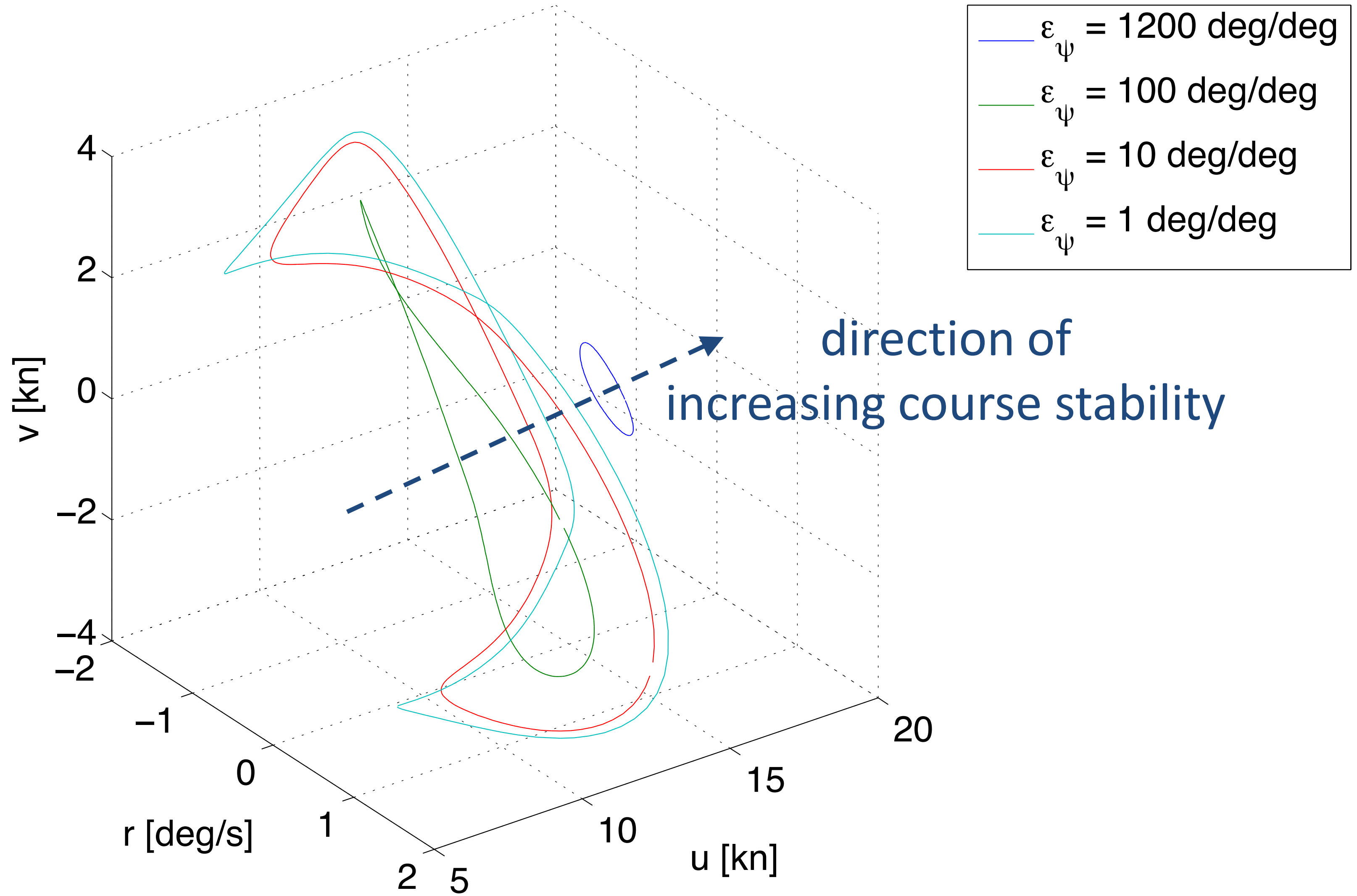
MANOEUVERING EQUATIONS - YAW CONTROL



ship velocities for several periodic solutions

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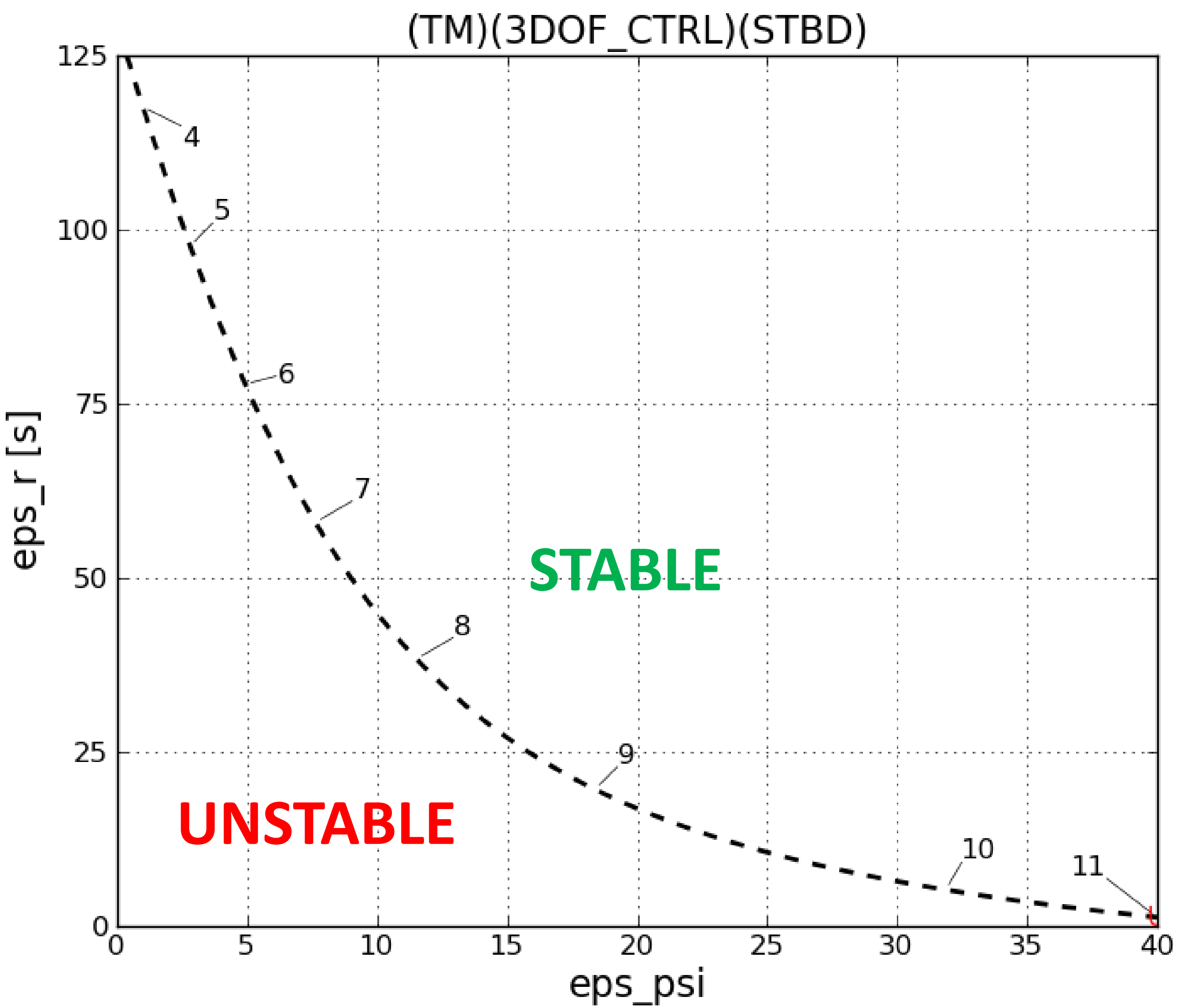
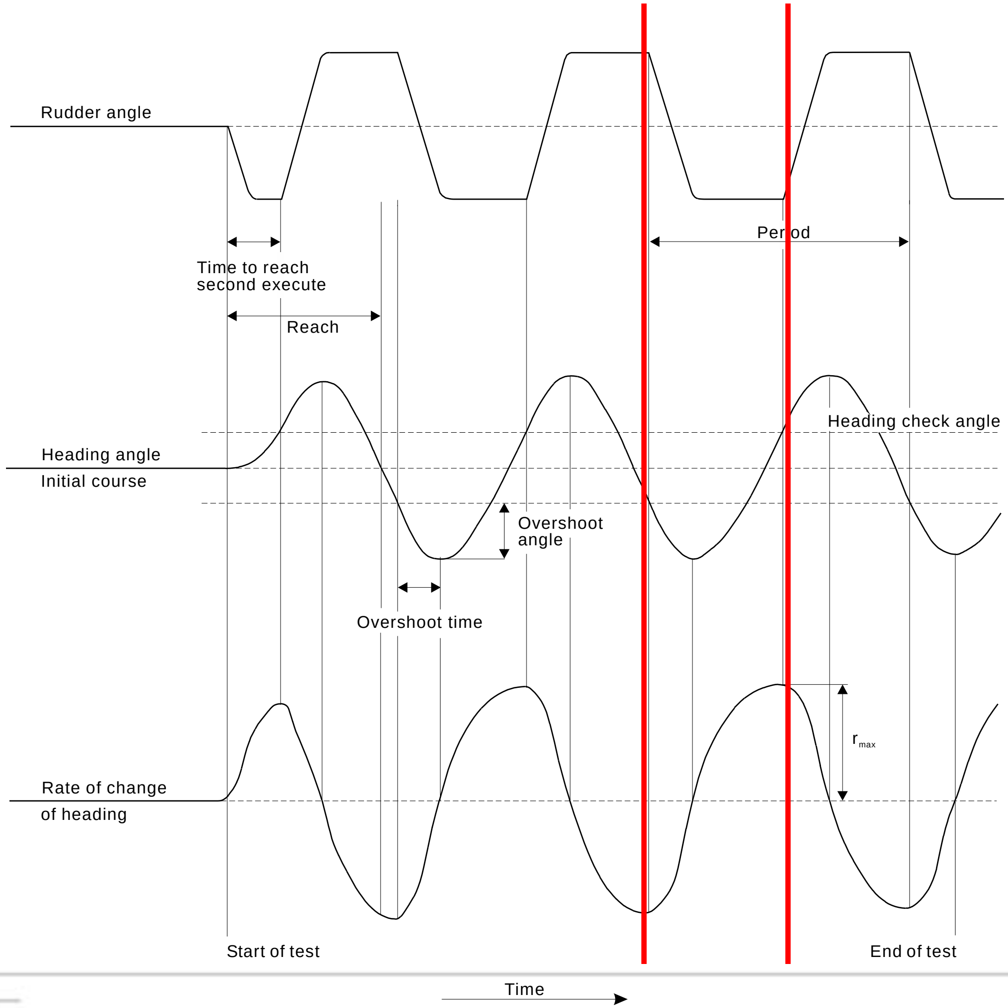
MANOEUVERING EQUATIONS - YAW CONTROL

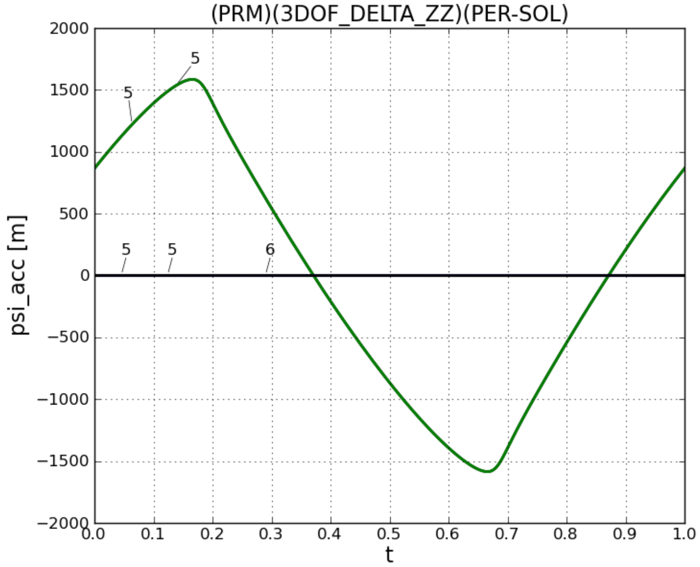


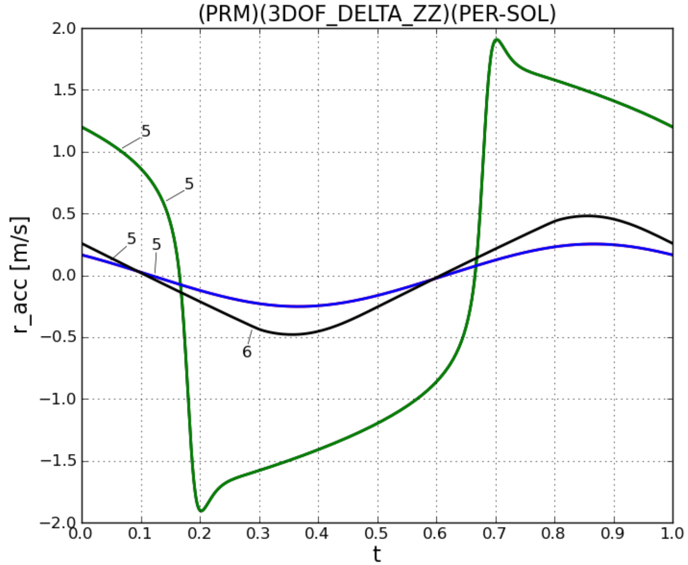
with AUTO it is easy to trace out the stability boundary …

MANOEUVERING EQUATIONS - ZIGZAG

consider half zigzag only and
use anti-symmetric boundary conditions







PARAMETRIC ROLLING

What happens if a container ship experiences large roll angles ?

PARAMETRIC ROLLING - SIMPLE ODE MODEL

simulation over large time intervals

$$(I + A)\ddot{\phi} + B(\dot{\phi})\dot{\phi} + C(t)\phi = 0 \qquad C(t) = \rho g V\left(GM + \partial GM(t)\right)$$

waves

wave force

$$h(t) = \sum_{j=1}^{n} h_j(t) = \sum_{j=1}^{n} \sqrt{2d(a,b)}\cos(\omega_j t + \gamma_j),$$

$$\partial GM(t) = \sum_{j=0}^{n} A_j \sqrt{2d(a,b)}\cos(\omega_j t + \gamma_j + \beta_j)$$

transfer coefficients for amplitude change and phase shift
of waves acting upon metacentric height: $A_j = A(\omega_j)$ $\beta_j = \beta(\omega_j)$

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exit time strategy:

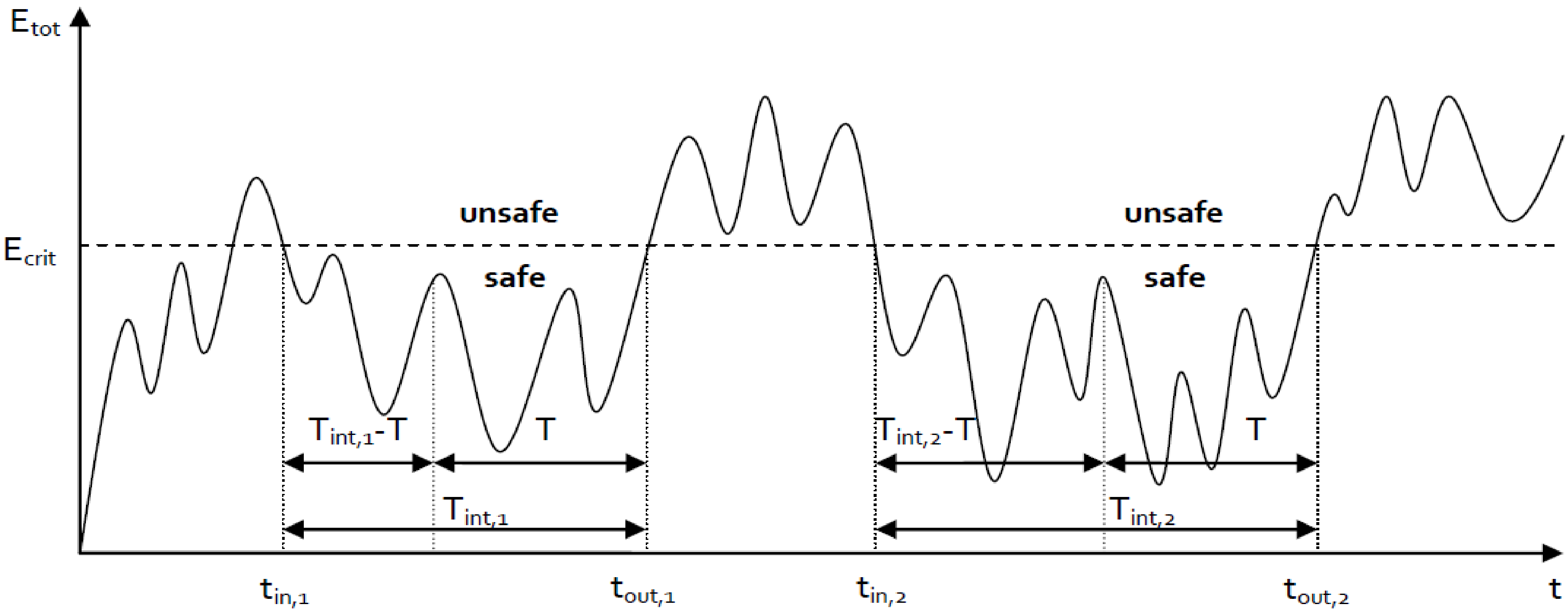
- very long time domain simulation

$$(I+A)\ddot{\phi}+B(\dot{\phi})\dot{\phi}+C(t)\phi=0 \qquad C(t)=\overline{C}+\rho g V \partial GM(t)$$

- observe energy $E(\phi,\dot{\phi})=\frac{1}{2}(I+A)\dot{\phi}^2+\frac{1}{2}\overline{C}\phi^2$

- define critical amplitude and critical energy $E_{\text{crit}}=\frac{1}{2}\overline{C}\varphi_{\text{crit}}^2$

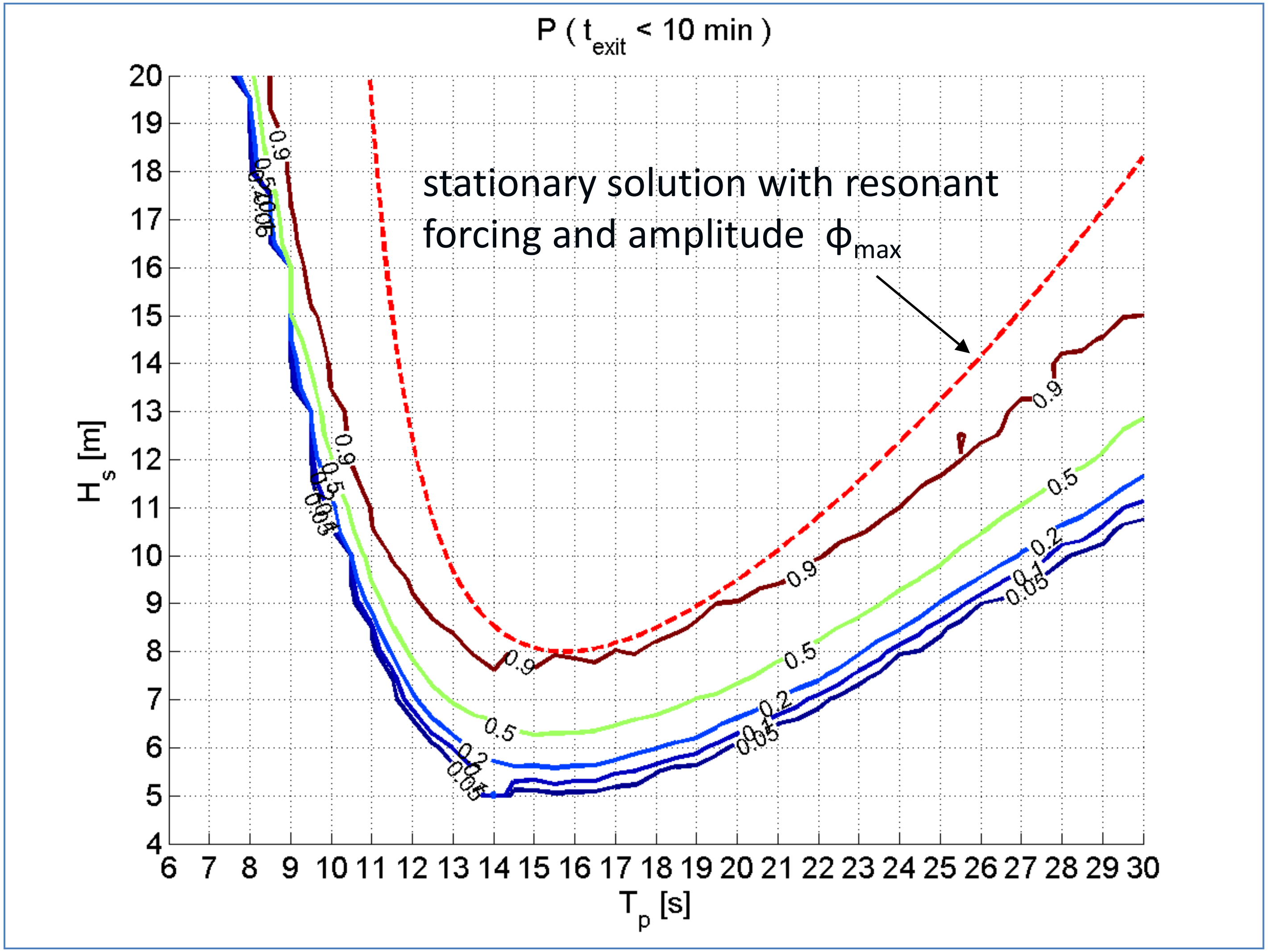
PARAMETRIC ROLLING - EXIT TIME STRATEGY



Fraction of runs arriving within time T at E_{crit}: $q(T) = min\left(\frac{T}{T_{int}}, 1\right)$

Weighted average over all safe zones: $\hat{q}(T)$

PARAMETRIC ROLLING - PROBABILITY



SHORT CRESTED WAVES

long crested waves are ‘easy’:

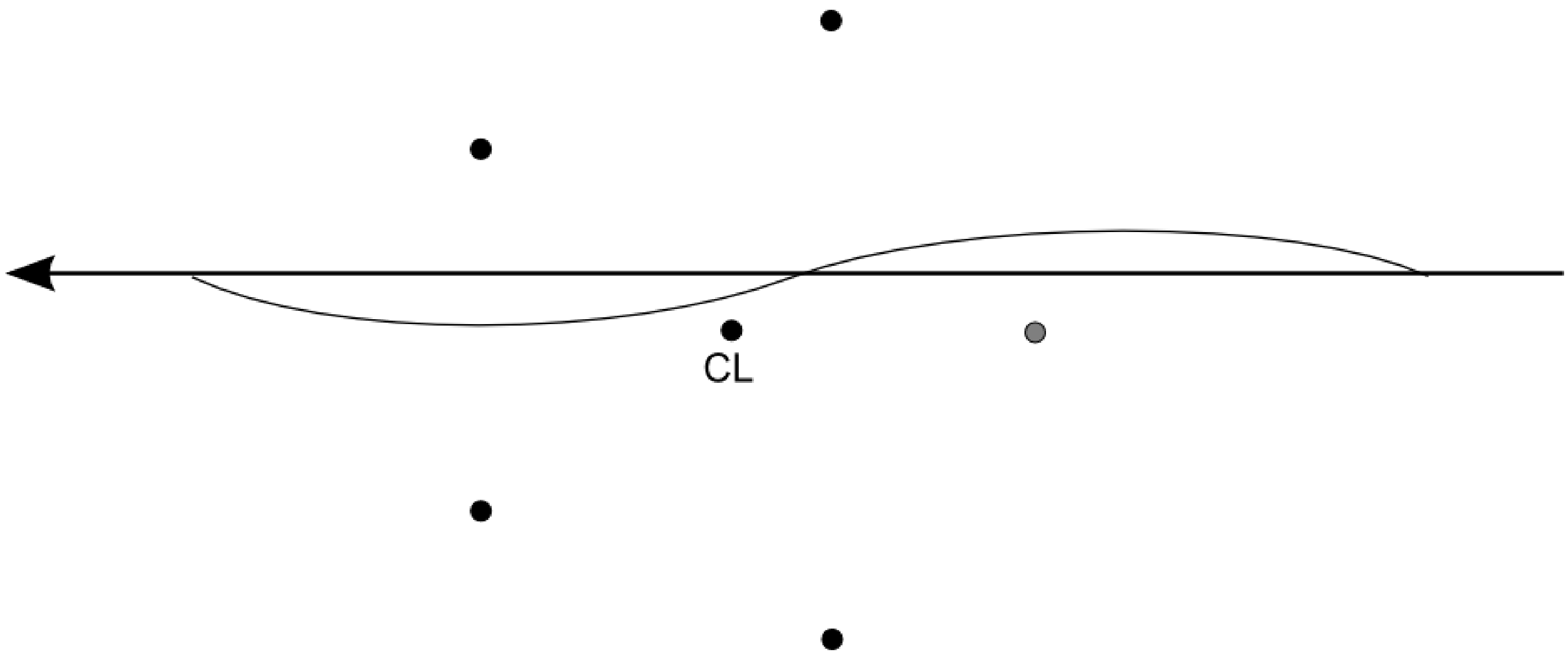
- to analyse
- to simulate

however:

real waves are short crested

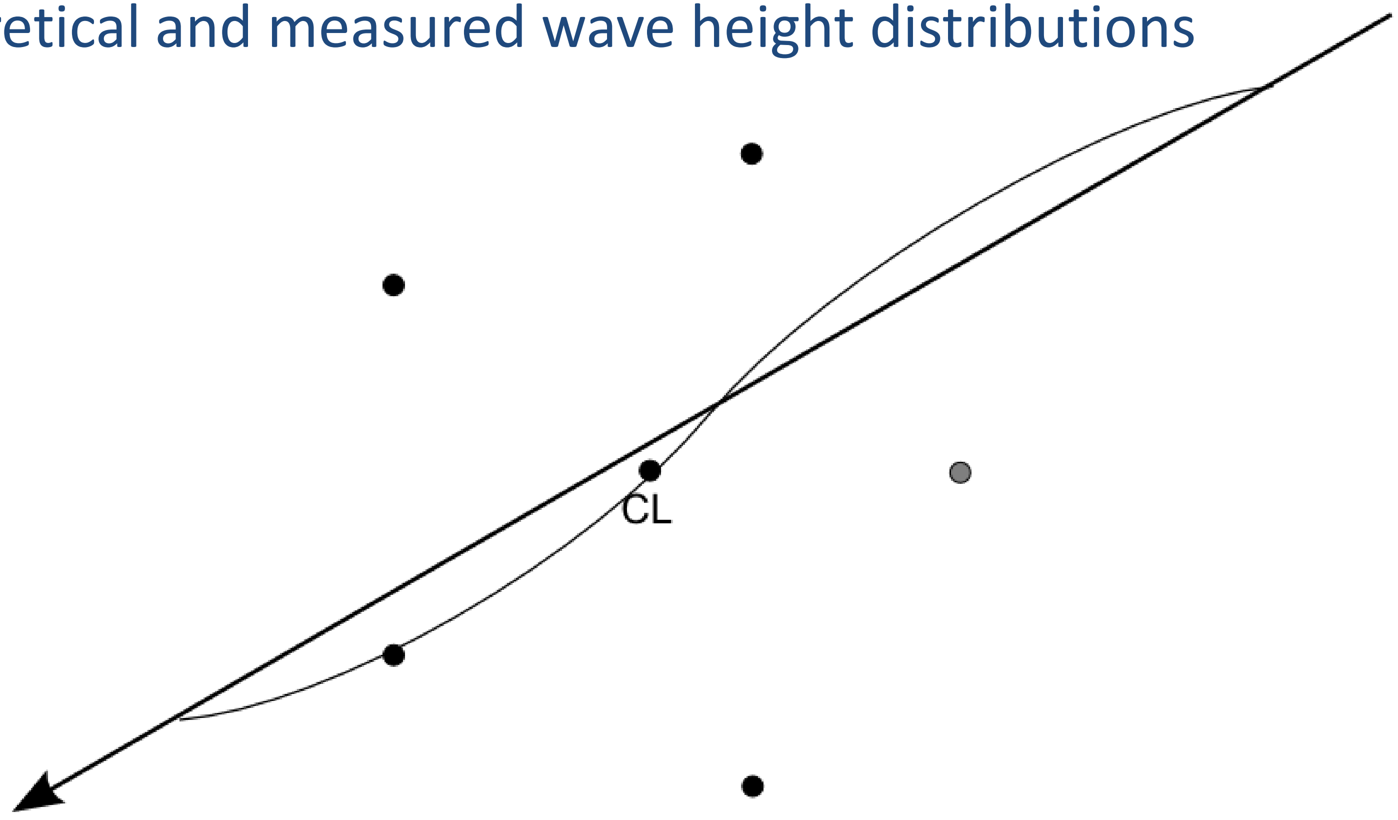
SHORT CRESTED WAVES

find wave spreading functions that match
theoretical and measured wave height distributions



SHORT CRESTED WAVES

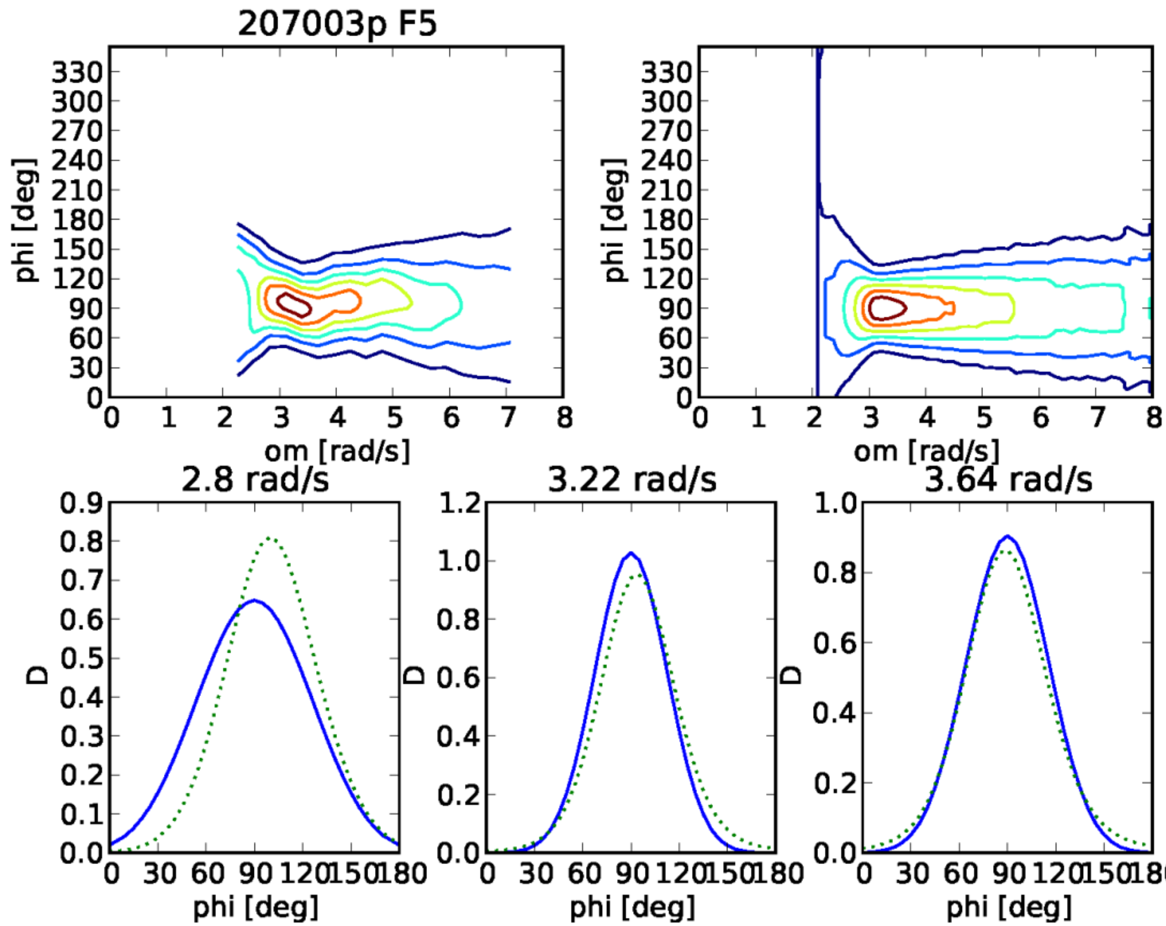
find wave spreading functions that match theoretical and measured wave height distributions



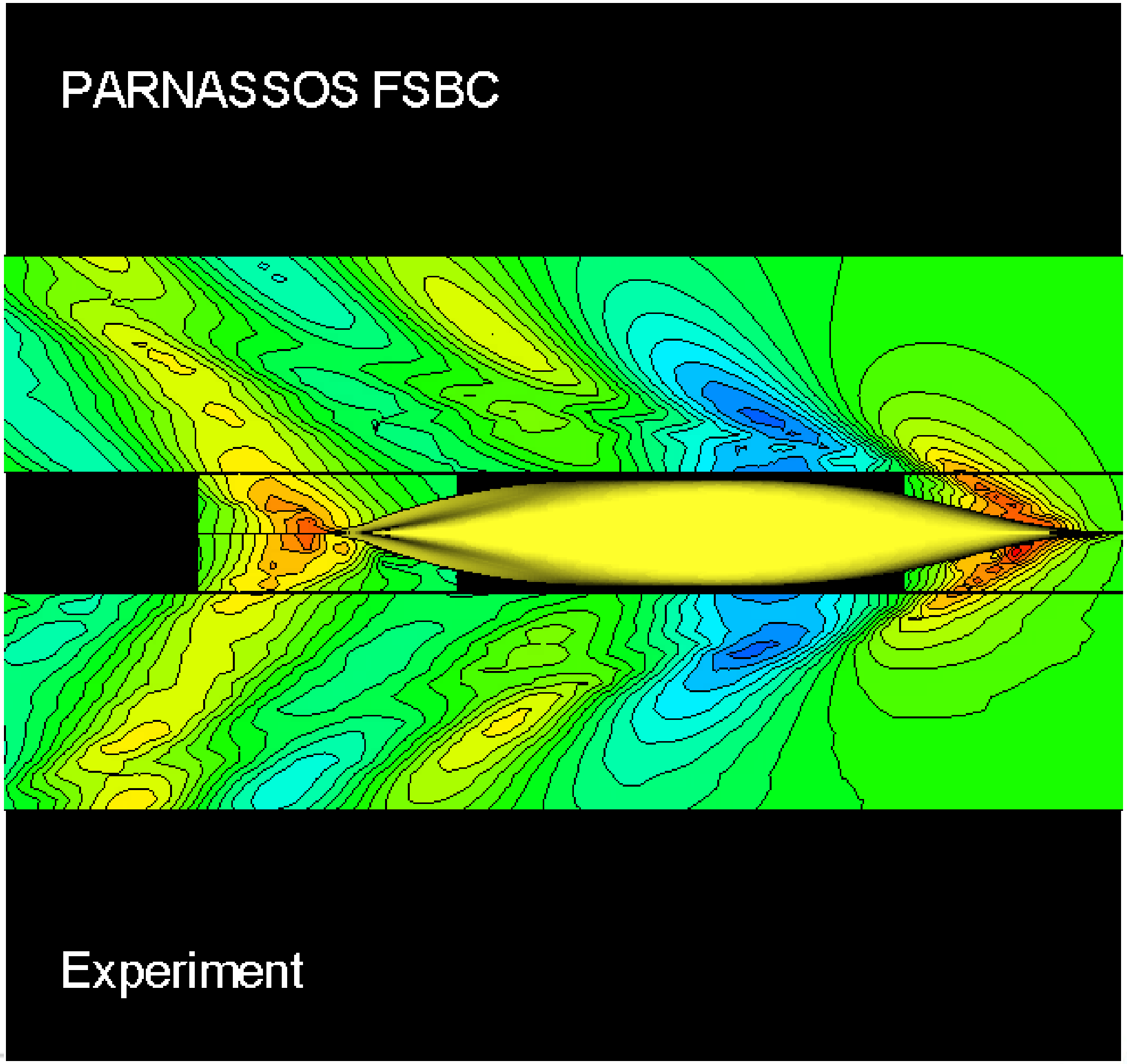
SHORT CRESTED WAVES

wave calibration using Maximum Likelihood Method

→ find wave spreading functions that match measured cross spectra with theoretical wave height transfer function



NUMERICAL DAMPING AND DISPERSION



NUMERICAL DAMPING AND DISPERSION

Consider plane surface waves on 2D uniform flow. Linearise in these waves. They are represented by Fourier components:

$$\bar{q} \equiv \begin{pmatrix} u \\ w \\ \psi \end{pmatrix} = \iint \begin{pmatrix} \hat{u} \\ \hat{w} \\ \hat{\psi} \end{pmatrix} e^{\imath kx + sz} dkds$$

Substitute this in the linearised and discretised RANS equations:

$$L_h \cdot \bar{q} = \iint \hat{L}_h(k,s) \cdot \hat{q} e^{\imath kx + sz} dkds = 0$$

L_h = RANS-operator, $\hat{L}_h$ = discrete Fourier symbol of this operator

Non-trivial solution only exists if determinant of $\hat{L}_h$ vanishes.

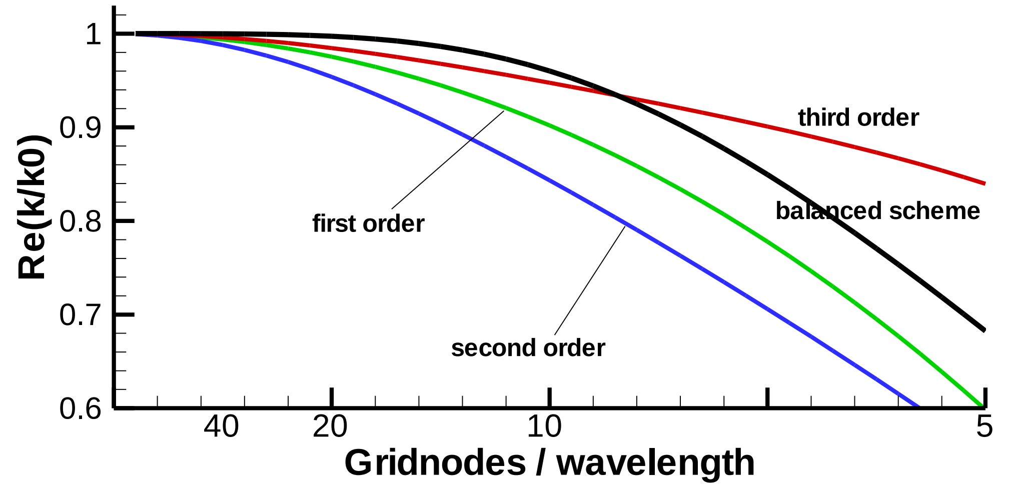
NUMERICAL DAMPING AND DISPERSION

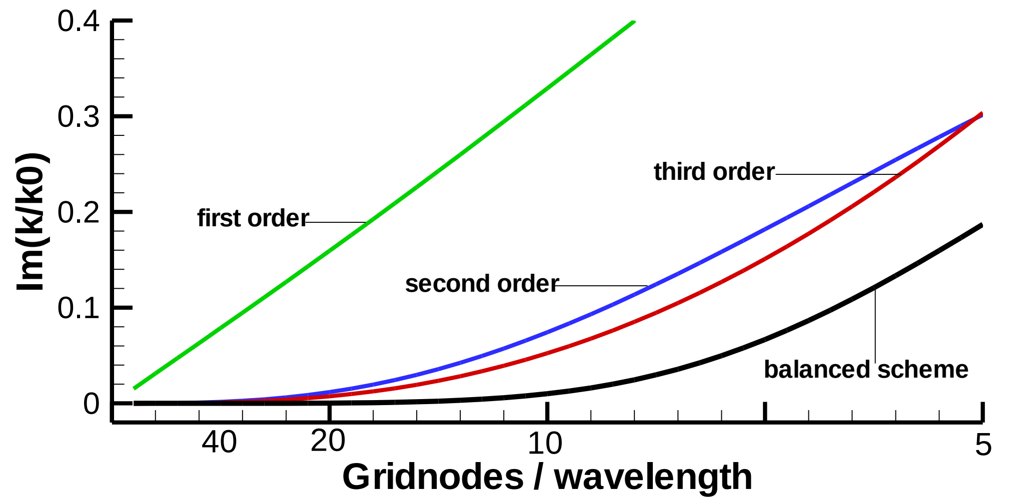
- Continuous problem: wave number $k_0 = 1/Fn^2$
- Discretised problem: wave number k, dependent on all difference schemes used.
- Damping and dispersion determined by k/k_0
- Dispersion determined by real part
- Damping determined by imaginary part

NUMERICAL DAMPING AND DISPERSION

- Standard:
  2^{nd}-order dispersion,
  3^{rd}-order damping

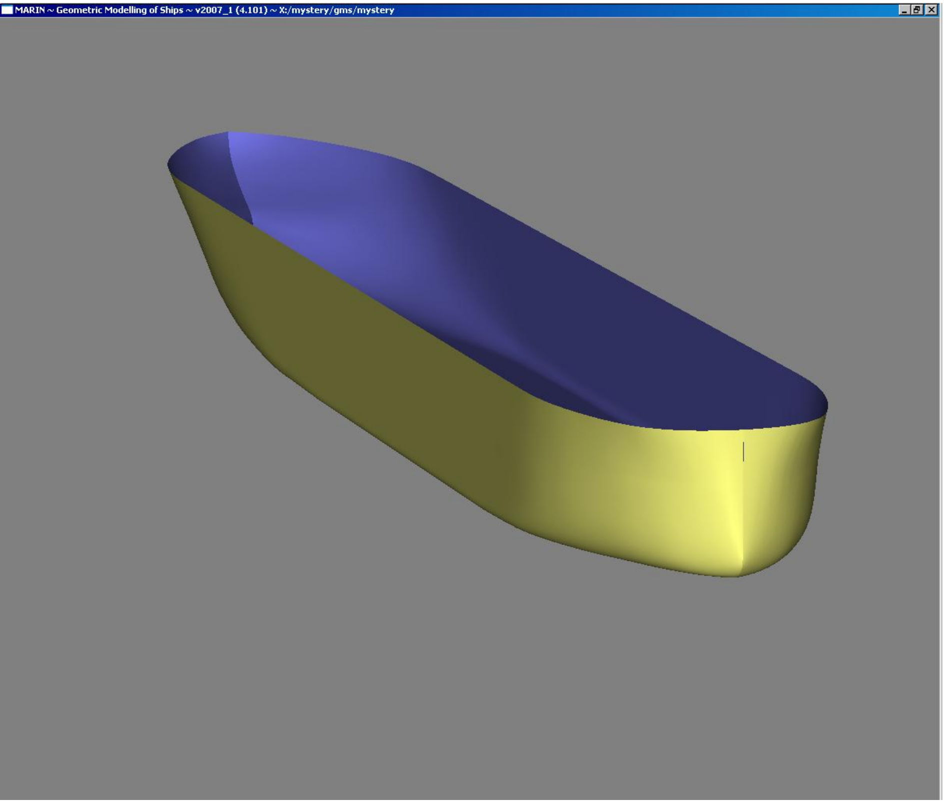
- It is possible to design a *dp/dx* scheme for the FSBC that cancels leading-order error terms from other difference schemes: 'Balanced scheme':
  3^{rd}-order dispersion,
  5^{th}-order damping

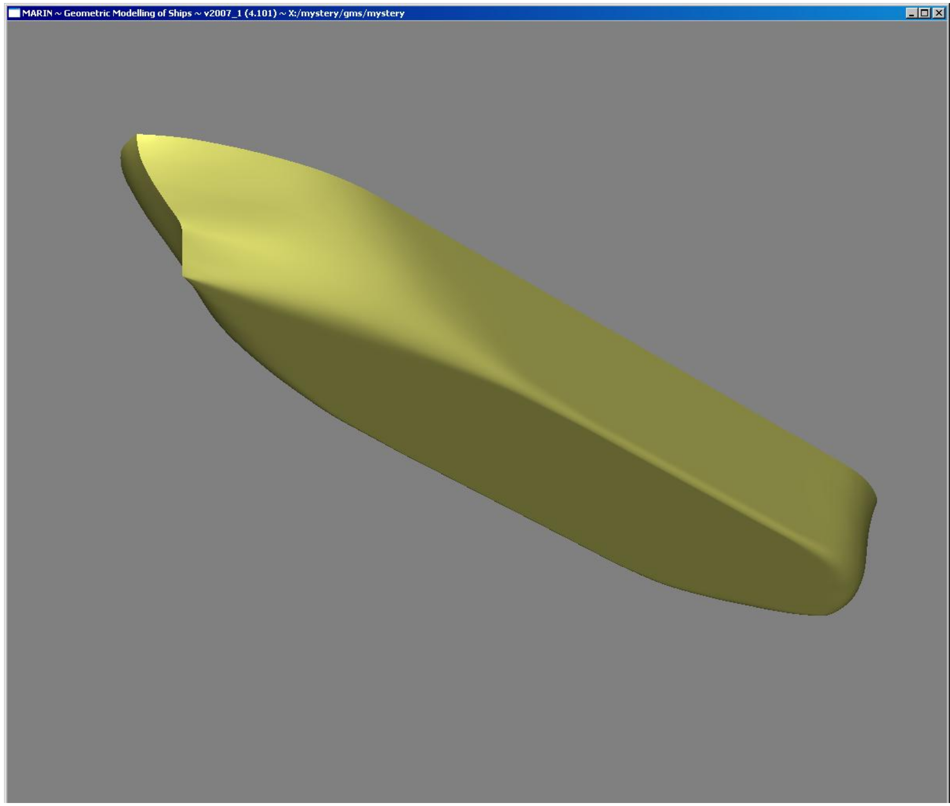




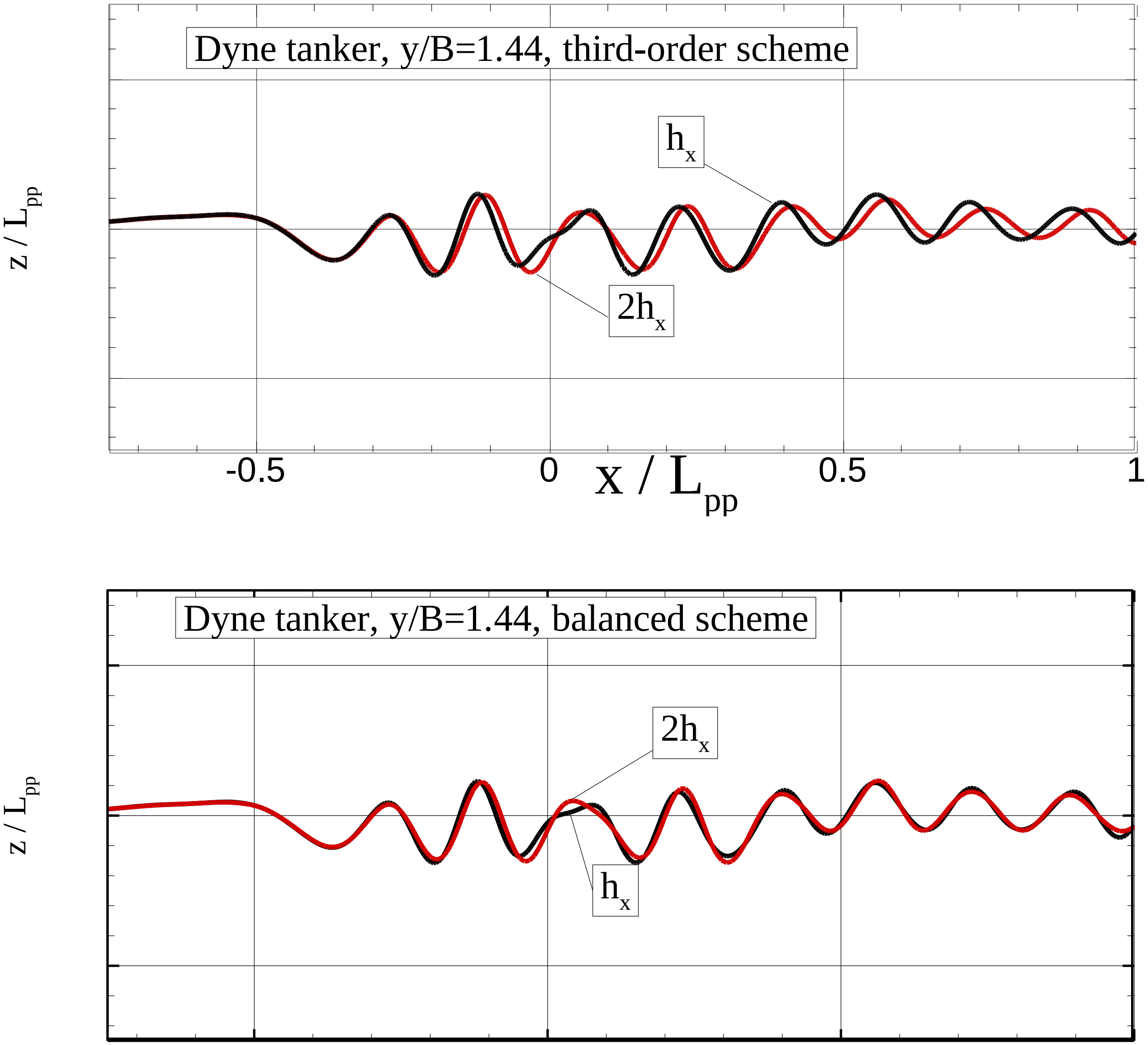
NUMERICAL DAMPING AND DISPERSION

Dyne tanker, Fn=0.165, Cb=0.87
model scale: 553x121x45 = 3.0M cells, full scale: 553x161x45 = 4.0M cells





NUMERICAL DAMPING AND DISPERSION

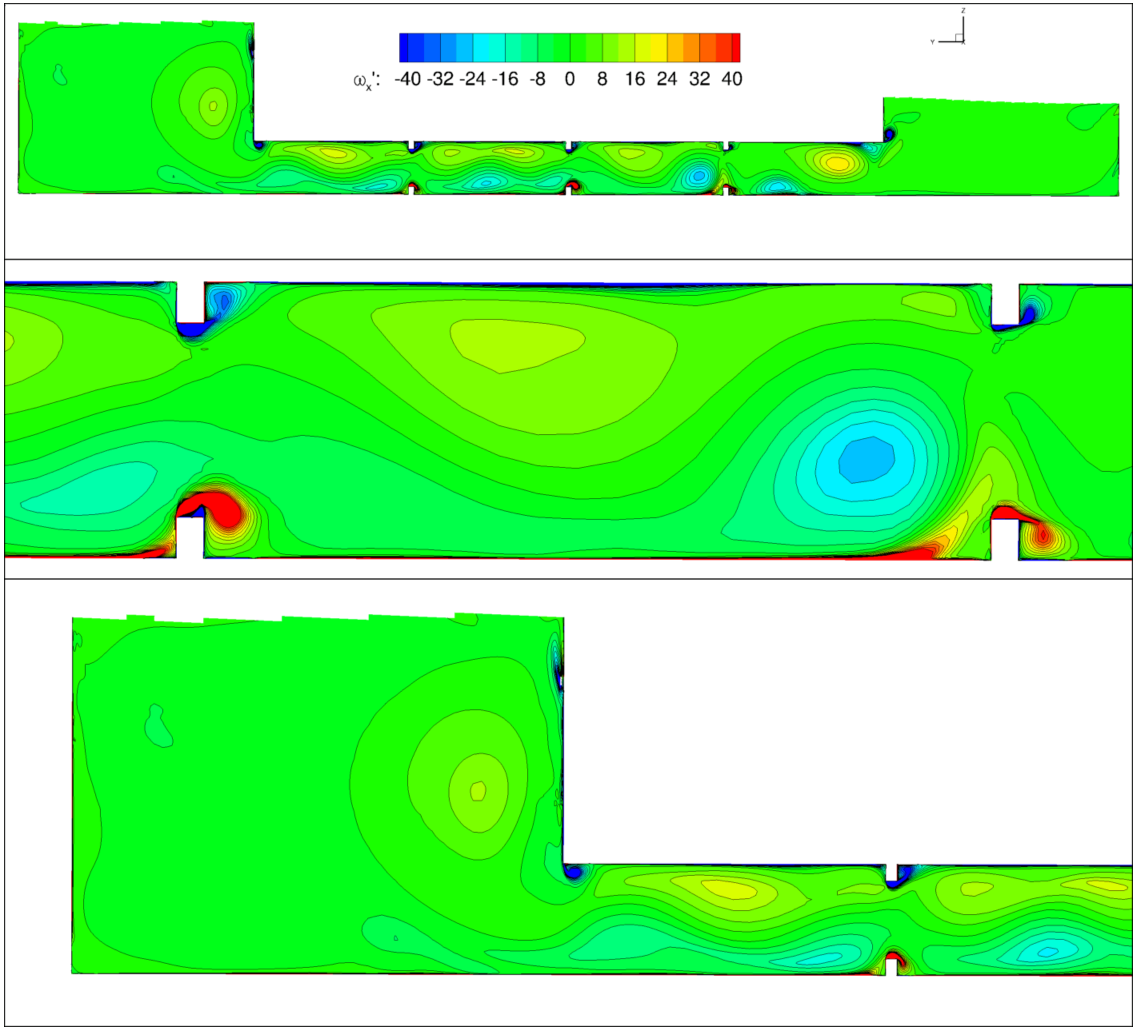


ANTI-ROLL TANKS

empty tank

filled tank

ANTI-ROLL TANKS – CFD U-TANK INTERNAL FLOW



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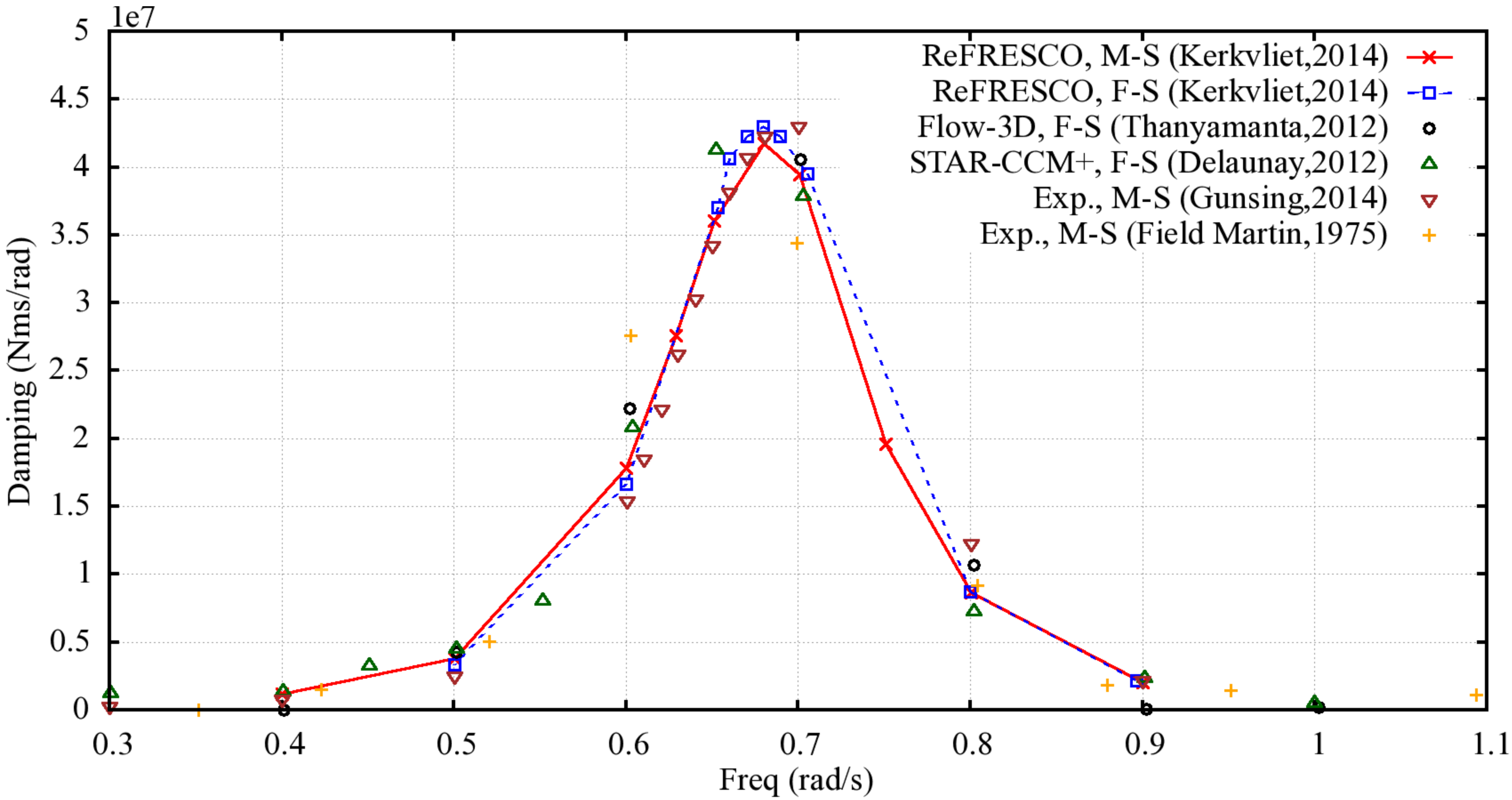
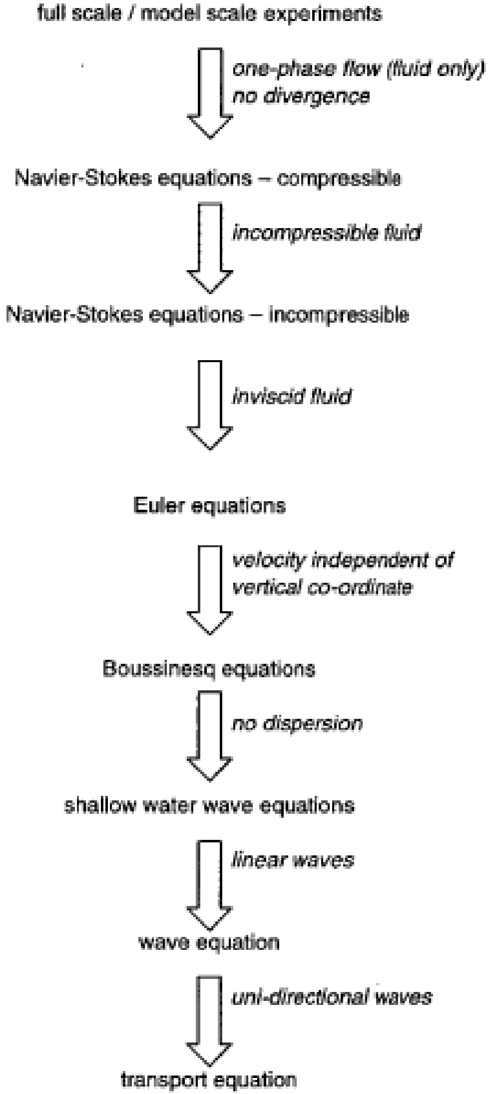
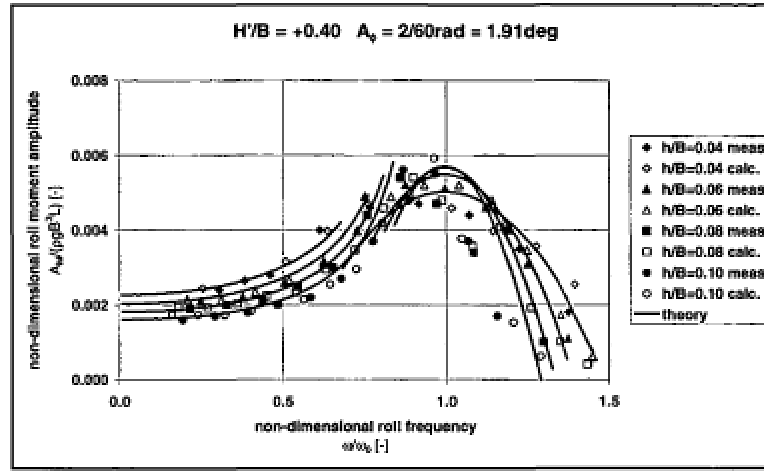
ANTI-ROLL TANKS – VALIDATION OF CFD



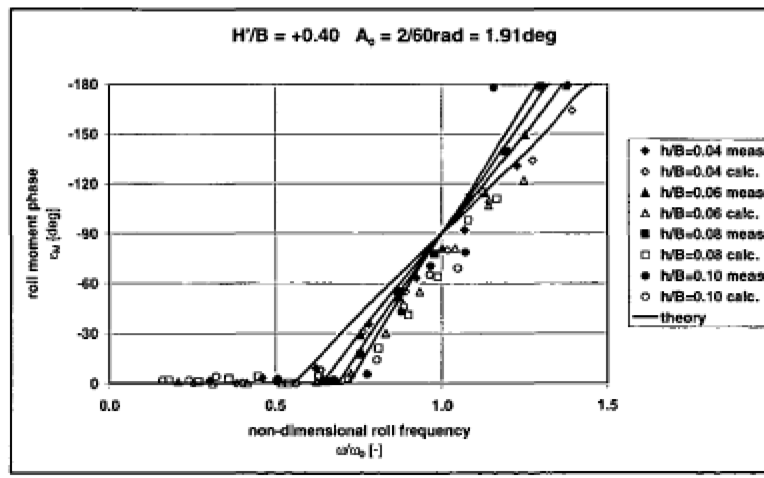
ANTI-ROLL TANKS - COMPLEX OR SIMPLE APPROACH ?



complex geometries: use CFD or experiments



simple geometries: use analytical-empirical models



REDUCTION OF OFFSHORE BASIN EFFECTS

- rectangular basin (44.8m x36m), adjustable depth up to 10 m
- individually controlled wave flaps on 2 sides (112/90)
- beaches on opposite sides
- wind and current

- due to the finite dimension of the basin, long crested waves are not entirely long crested
- reflections due to presence of test models (ships/offshore platforms) may affect test results

REDUCTION OF OFFSHORE BASIN EFFECTS

linearized potential flow model

$$\Delta\phi = 0$$

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0$$

$$\frac{\partial\phi}{\partial n} = \vec{V}\bullet\vec{n}$$

frequency domain

$$F(t)=\hat{F}e^{-i\omega t}$$

numerical solution:
boundary element (panel)
method with zero speed
Green functions G(P,Q)

$$\phi^j(P) = \int_S \sigma(Q)G(P,Q)dS$$

$$\frac{\partial\phi^j(P)}{\partial n} = 2\pi\sigma\left(P\right) + \int_S \sigma(Q)\frac{\partial G(P,Q)}{\partial n}dS$$

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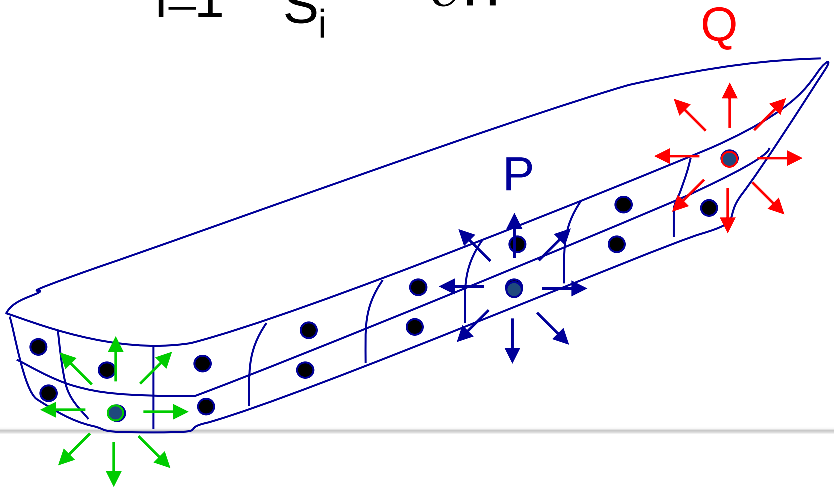
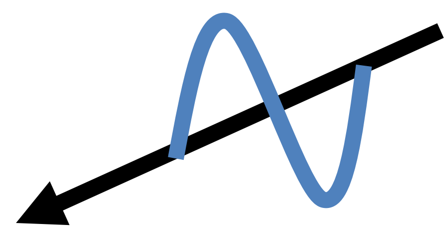
REDUCTION OF OFFSHORE BASIN EFFECTS

discretization: constant source per panel

$$\phi^j(P) = \sum_{i=1}^{N} \sigma_i \int_{S_i} G\left(P,Q\right) dS$$

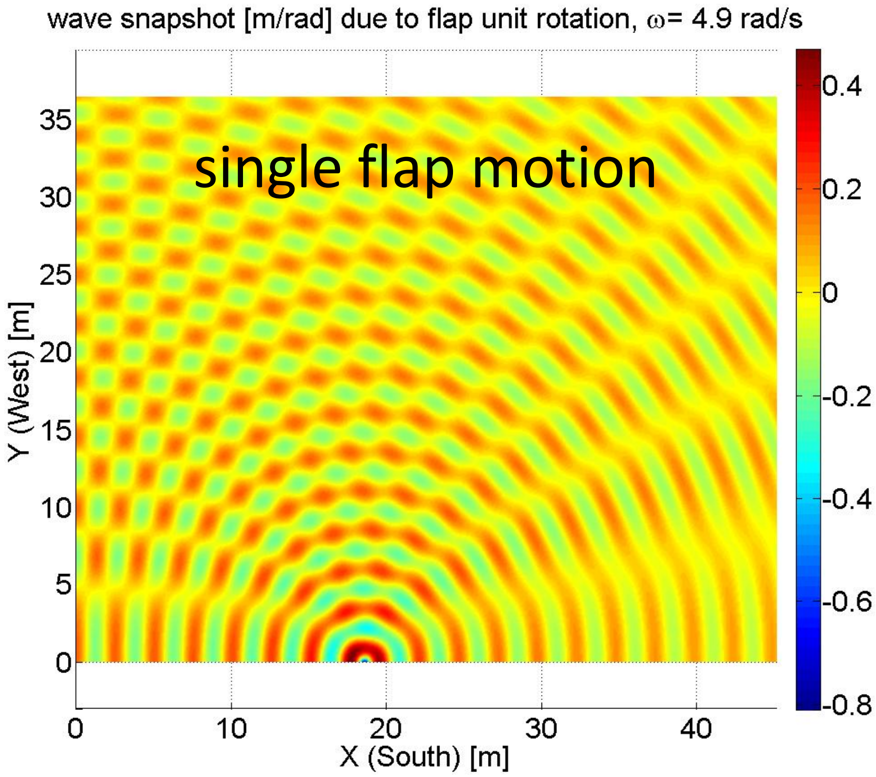
$$\frac{\partial \phi^j(P)}{\partial n} = 2\pi\sigma\left(P\right) + \sum_{i=1}^{N} \sigma_i \int_{S_i} \frac{\partial G(P,Q)}{\partial n} dS$$





REDUCTION OF OFFSHORE BASIN EFFECTS

use ship motion program to calculate basin waves (why not?)
waves generated by flaps
each flap is modeled as a separate moving body



linear superposition of waves

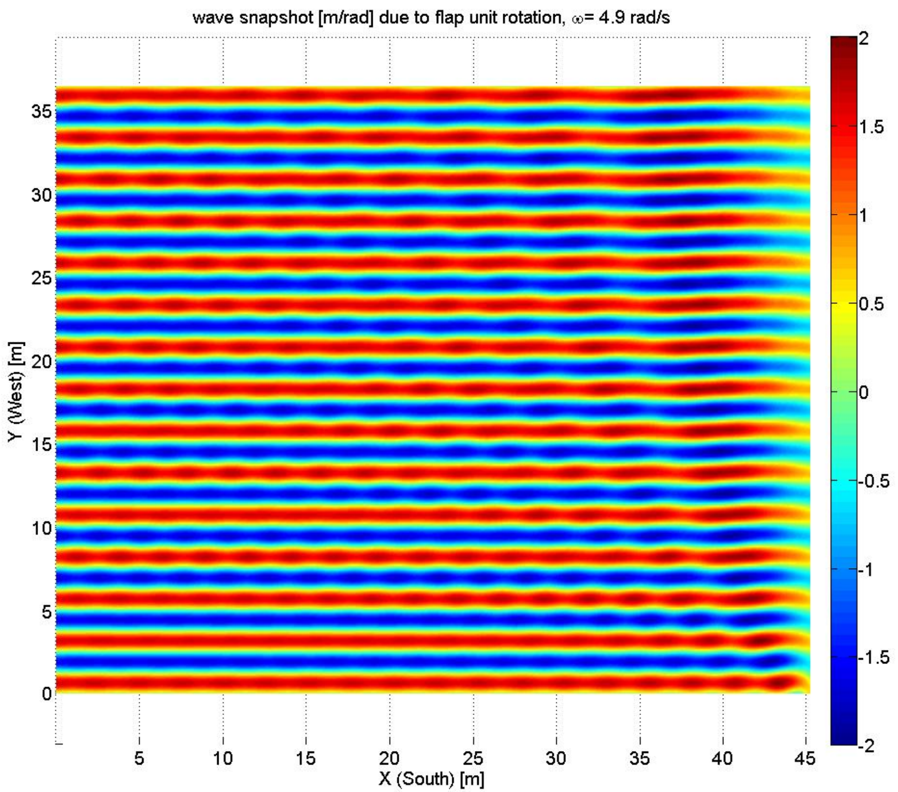
$$\eta_{tot}(x,y,\omega) = \sum_{k=1}^{Npaddle} A_k \eta_k(x,y,\omega)$$

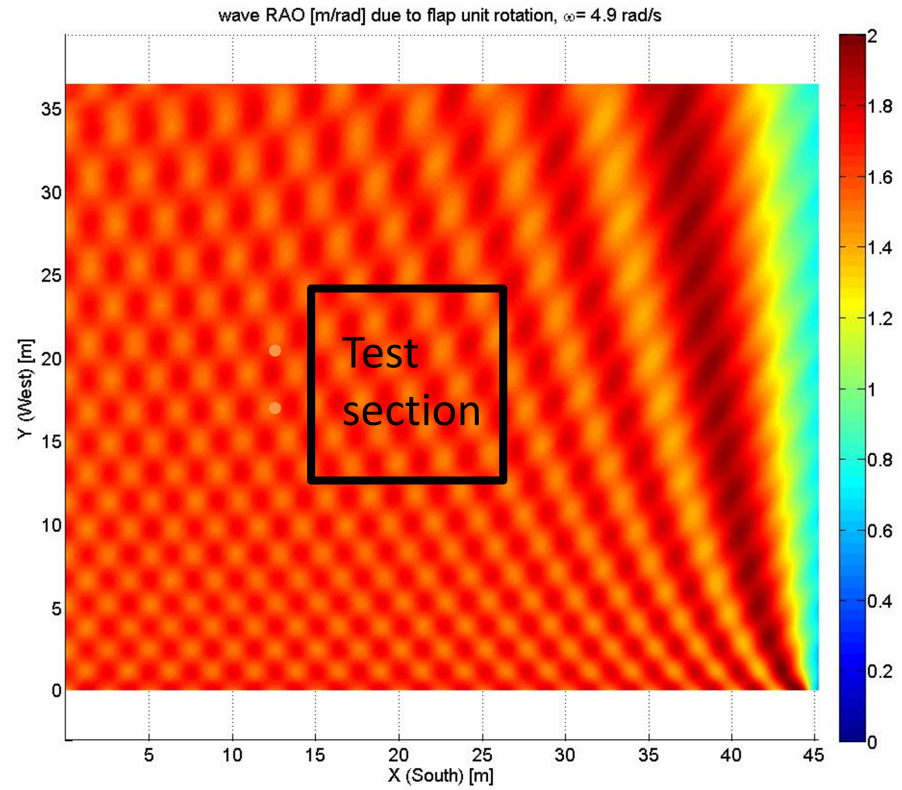
beaches are modeled as open boundaries (no reflections)

REDUCTION OF OFFSHORE BASIN EFFECTS

waves generated by all flaps on south side moving identically

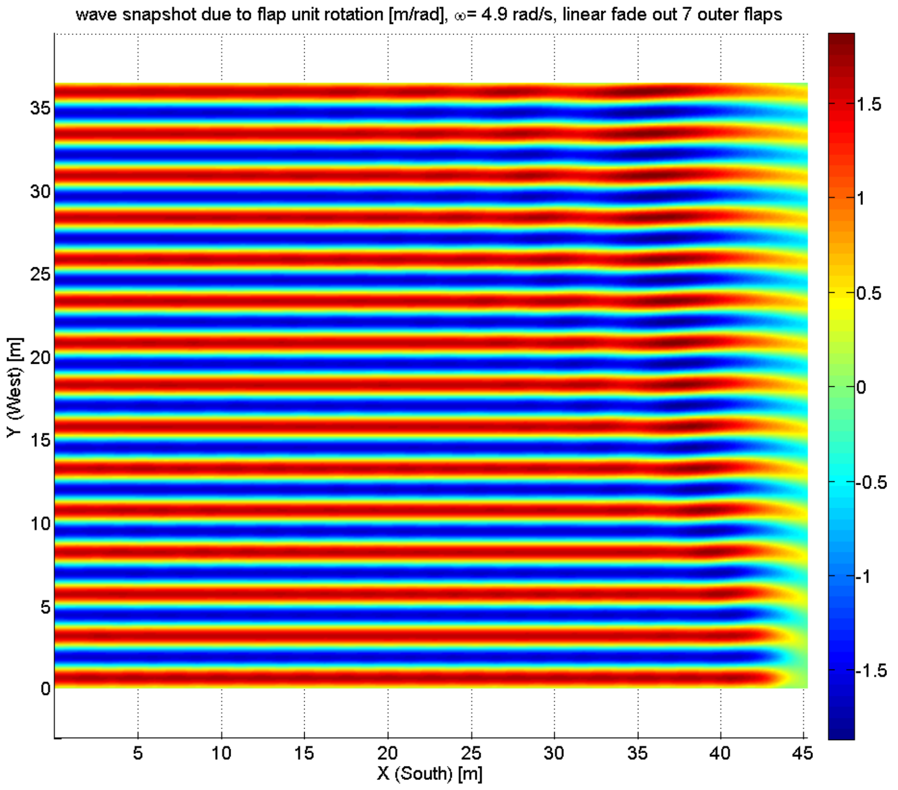
→ this is not long-crested !!!

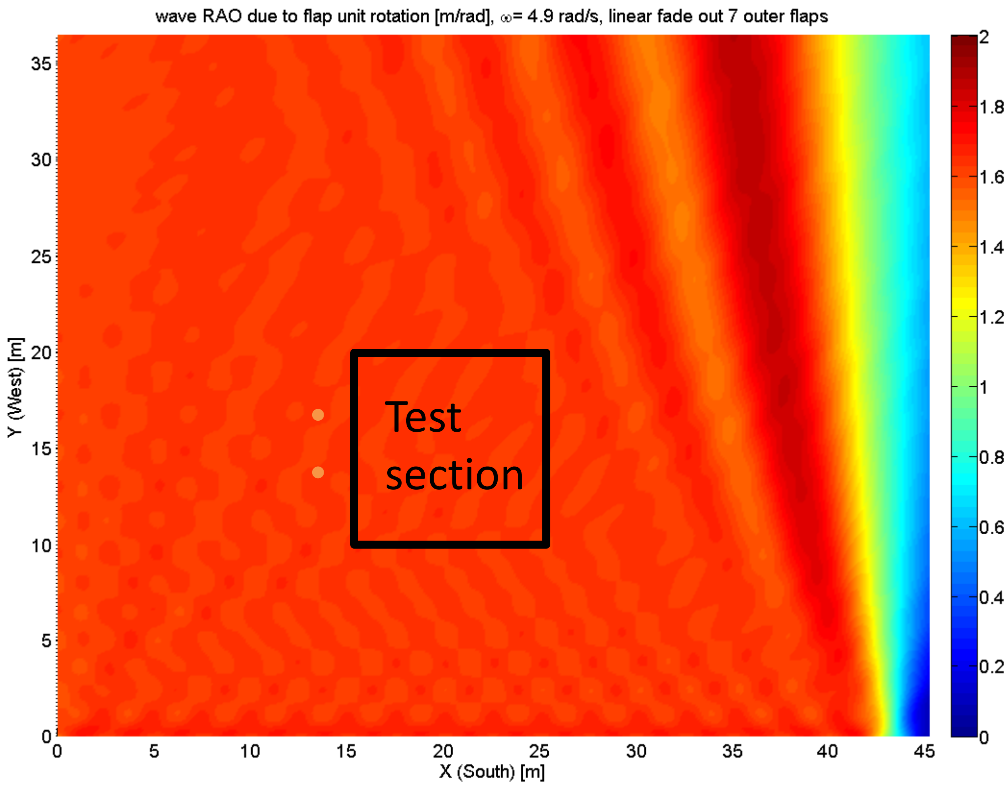




REDUCTION OF OFFSHORE BASIN EFFECTS

linear amplitude fade-out of outer 7 flaps
(default solution in basin wave control software)





→ already much better !!!

REDUCTION OF OFFSHORE BASIN EFFECTS

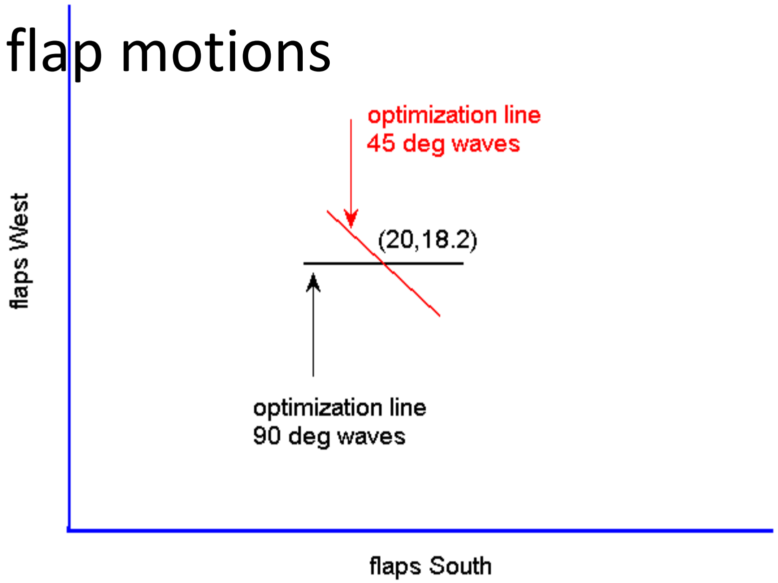
Can we do better than linear fade-out? Use optimization techniques!

Minimize the objective function $$F(A_1 \dots A_M) = \sqrt{\sigma(\eta_{tot,r})^2 + \sigma(\eta_{tot,i})^2}$$

$\eta_{tot,r}$ real part total wave elevation on optimization line

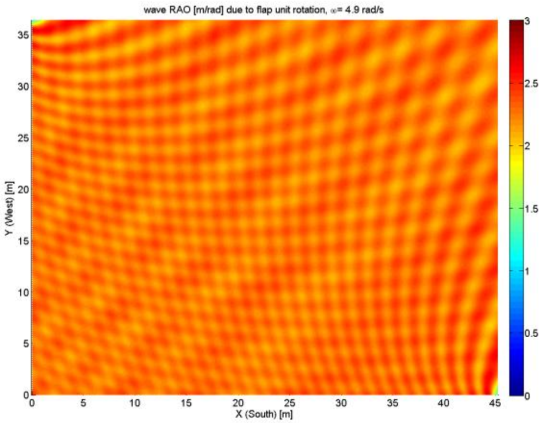
$\eta_{tot,i}$ imaginary part total wave elevation on optimization line

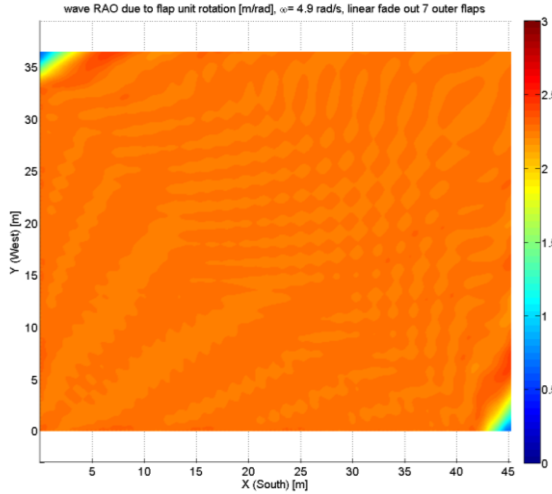
$A_1 \dots A_M$ complex amplitudes of M flap motions

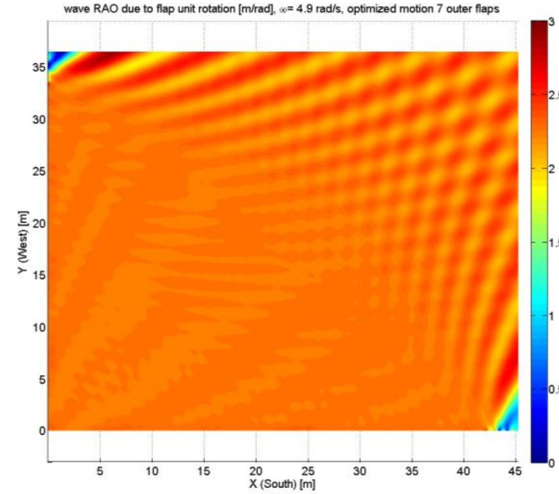


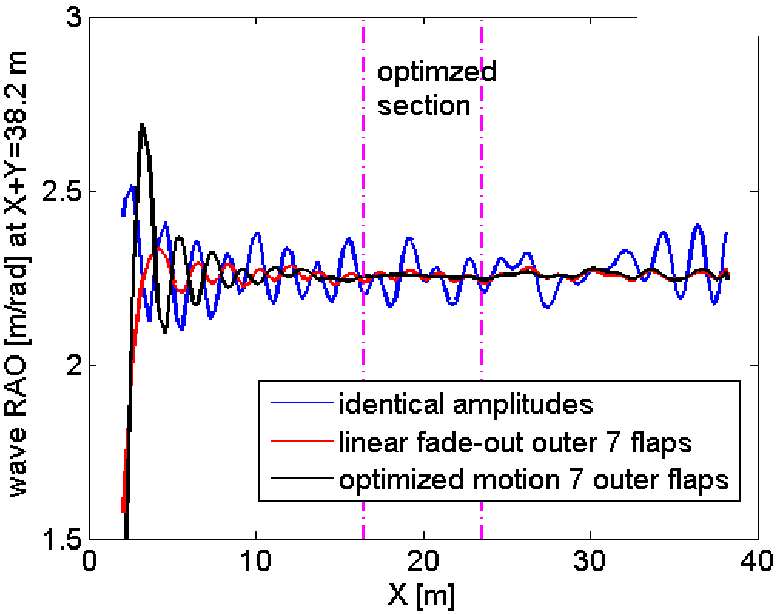
REDUCTION OF OFFSHORE BASIN EFFECTS

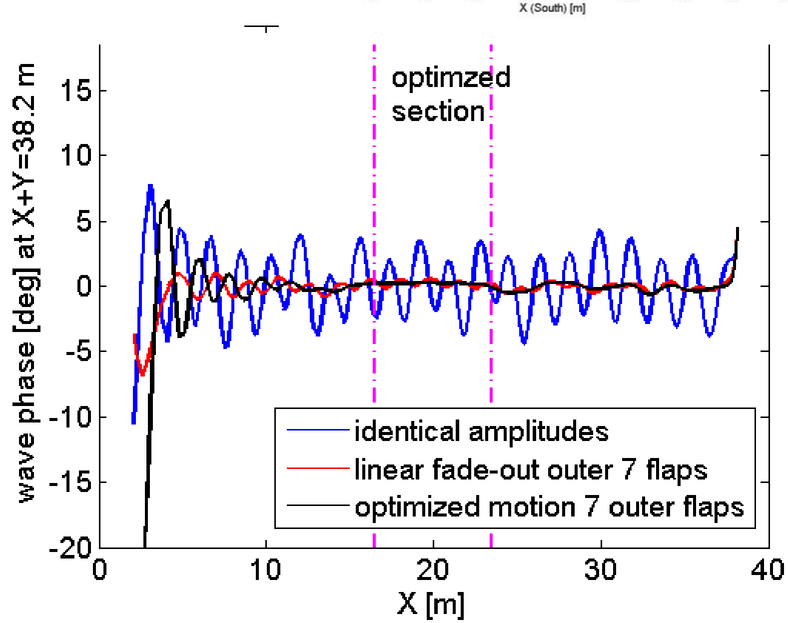
45 degrees waves





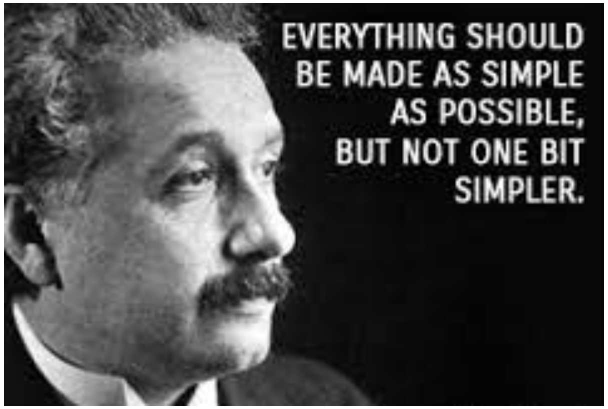






CONCLUSIONS

- applied mathematics is *essential* for research on ship hydromechanics
- successful application of mathematics requires knowledge of mathematical solution techniques *and* understanding of physical / technical problems
- we need mathematicians that can think/talk/do (ship) hydromechanics *and* naval architects* that can think/talk/do (applied) mathematics, both at a sufficient level to ‘reach out and touch’
- we need ability and courage to model:
  - reduce & assume
  - extrapolate & validate
  - think ‘out of the box’



* and people from other disciplines, of course

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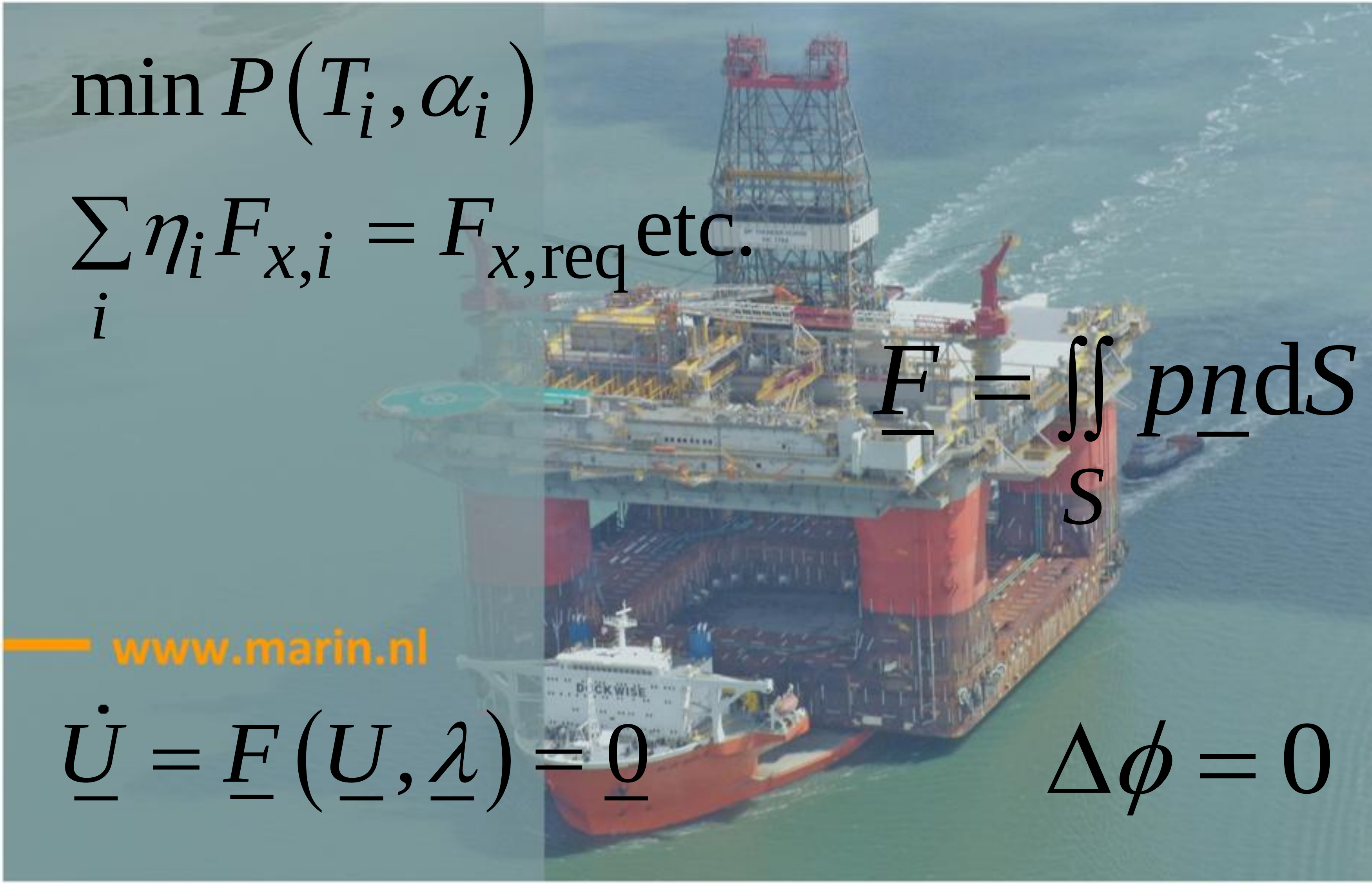
THANK YOU FOR YOUR ATTENTION !



$$\min P\left(T_i, \alpha_i\right)$$

$$\sum_i \eta_i F_{x,i} = F_{x,\mathrm{req}} \; \text{etc.}$$

$$\underline{F} = \iint_S p\underline{n}\,\mathrm{d}S$$

$$\dot{\underline{U}} = \underline{F}\left(\underline{U}, \underline{\lambda}\right) = \underline{0}$$

$$\Delta\phi = 0$$

www.marin.nl

MARIN
P.O. Box 28
6700 AA Wageningen
The Netherlands
T +31 317 49 39 11
F +31 317 49 32 45
E info@marin.nl
I www.marin.nl