More efficient models and better understanding of Risks in Finance

L.A. Grzelak,

"*Models which are more realistic are often not used in practice not because of ignorance but because of insufficient numerical efficiency and lack of transparency*"

3TU.AMI

Utrecht

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Short about Lech A. Grzelak

- 2004-2006: MSc in Risk Modeling; Thesis on modeling of corrosion in Gas pipelines (Gasunie);
- 2004-2006: MSc in Computer Science and Econometrics; Thesis on modeling of electricity prices with heavy tail distributions;
- 2006-2007: Quantitative Energy Analyst at Saen Energy (supporting trading desk with tools for trading energy derivatives, mainly electricity);
- 2007-2011: PhD in Applied Mathematics, "Equity and Foreign Exchange Hybrid Models for Pricing Long-Maturity Financial Derivatives" TU Delft;
- 2011-2014: Post-doc Scientist at CWI (0.2fte);
- 2011-: Sr. Quantitative Analyst at Rabobank, developing pricing models for Inflation, FX, Interest rates and Equity;
- 2014- Lecturer/Docent at TUDelft (0.2fte); Course on Financial Engineering;

Teaching:

- Special topics in Financial Engineering - the course is focused on recent developments in modeling.

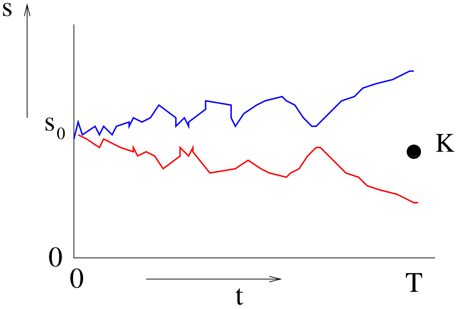
Main research topics

- Accurate and efficient pricing of complex financial derivatives (the so-called exotics).
- Focus on models used for long-term investments (e.g. pension funds).
- Developing models with multiple sources of uncertainty (multidimensional SDEs and hybrid models).
- Developing models which do not require calibration (stochastic-local volatility models).
- Numerical techniques for improving Monte Carlo simulation (the collocation method) and simulation of complex financial models.

Financial derivatives

Call options

An option gives the holder the right to trade in the future at a previously agreed strike price, K.



$V(T,S) = \max(S_T - K, 0) =: h(S_T)$

Feynman-Kac Theorem (option pricing formulation)

Given the option valuation problem

$$\begin{cases} \dfrac{\partial V}{\partial t} & + \quad \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + rS\dfrac{\partial V}{\partial S} - rV = 0, \\ V(T,S) & = \quad h(S_T) = \text{ given} \end{cases}$$

Then the value, $V(t,S)$, is the unique solution of

$$V(t,S) = e^{-r(T-t)}\mathbb{E}^Q\{V(T,S_T)|\mathcal{F}_t\}$$

with the sum of first derivatives square integrable, and $S = S_t$ satisfies the system of SDEs:

$$dS_t = rS_t dt + \sigma S_t d\omega_t^Q,$$

- Similar relations also hold for (multi-D) SDEs and PDEs!

A pricing approach; European options

$$V(t_0, S_0) = e^{-r(T-t_0)}\mathbb{E}^Q\{V(T, S_T)|\mathcal{F}_0\}$$

Quadrature:

$$V(t_0, S_0) = e^{-r(T-t_0)} \int_{\mathbb{R}} V(T, S_T) f(S_T, S_0) dS_T$$

- Trans. PDF, $f(S_T, S_0)$, typically not available.

Increasing dimensions: Multi-asset options

- The problem dimension increases if the option depends on more than one asset S_i (multiple sources of uncertainty).
- If each underlying follows a geometric (lognormal) diffusion process, where the correlation of each asset to all other assets is constant:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sum_{i,j=1}^{d}[\sigma_i\sigma_j\rho_{i,j}S_iS_j\frac{\partial^2 V}{\partial S_i\partial S_j}] + \sum_{i=1}^{d}[(r-\delta_i)S_i\frac{\partial V}{\partial S_i}] - rV = 0 \ .$$

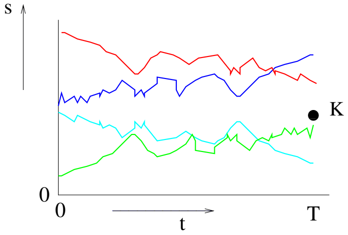
- Each additional asset is represented by an extra dimension in the problem:
- Required information is the volatility of each asset σ_i and the correlation between each pair of assets $\rho_{i,j}$.

Multi-asset options

Multi-asset options belong to the class of exotic options.

$$V(\mathbf{S}, T) = \max(\max\{S_1, \ldots, S_d\}_T - K, 0) \text{ (max call)}$$

$$V(\mathbf{S}, t_0) = e^{-r(T-t_0)} \int_{\mathbb{R}} V(\mathbf{S}, T) f(\mathbf{S}(T)|\mathbf{S}(t_0)) d\mathbf{S}$$



$\Rightarrow$ High-dimensional integration techniques.

Financial industry; Work at Banks

- Financial engineering, post-crisis pricing approach:
  1. Start with some financial product
  2. Model asset prices involved (SDEs)
  3. Calibrate the model to market data (Numerics, Opt.)
  4. Model product price correspondingly (PDE, Integral)
  5. Price the product of interest (Numerics, MC)
  6. Set up a hedge to remove the product risk (Opt.)

Using of Stochastic Interest Rates

- When pricing of long-maturity derivatives it is important to account for stochastic nature of interest rates.
- Inclusion of additional risk-factors is straightforward, the problem is to perform efficient calibration.
- An example is the Heston-Hull-White hybrid:

$$
\begin{aligned}
\mathrm{d}S(t) &= r(t)S(t)\mathrm{d}t + \sqrt{v(t)}\mathrm{d}W_1(t),\\
\mathrm{d}v(t) &= \kappa(\bar{v} - v(t))\mathrm{d}t + \gamma\sqrt{v(t)}\mathrm{d}W_2(t),\\
\mathrm{d}r(t) &= \lambda(\theta(t) - r(t))\mathrm{d}t + \eta\mathrm{d}W_3(t),
\end{aligned}
$$

$\mathrm{d}W_1(t)\mathrm{d}W_2(t) = \rho_{1,2}\mathrm{d}t$, $\mathrm{d}W_1(t)\mathrm{d}W_3(t) = \rho_{1,3}\mathrm{d}t$, $\mathrm{d}W_2(t)\mathrm{d}W_3(t) = \rho_{2,3}\mathrm{d}t$.

- When assuming stochastic interest rates we need to handle joint distribution of the rates and the underlying asset:

$$
\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s} V(T,S)\right] \neq \mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s}\right]\mathbb{E}^{\mathbb{Q}}\left[V(T,S)\right]
$$

Stochastic Local Volatility Models

- Non-parametric local volatility models enable perfect calibration without additional optimization routines.
- The key-element is that one of the parameters is expressed in terms of the market instruments.
- The Heston Stochastic Local Volatility (SLV) Model:

$$\begin{aligned}\frac{\mathrm{d}S(t)}{S(t)} &= r\mathrm{d}t + \boxed{\sigma(t,S(t))}\sqrt{v(t)}\mathrm{d}W_S(t),\\ \mathrm{d}v(t) &= \kappa(\bar{v}-v(t))\mathrm{d}t + \gamma\sqrt{v(t)}\mathrm{d}W_v(t)\end{aligned}$$

- $\sigma(t,S(t))$ is chosen such that it acts as a **compensator** that bridges the gap between the (poorly) calibrated Heston model and the market.
- How should $\sigma(t,S(t))$ be chosen? Intuition: it should depend on market data.

Stochastic Local Volatility Models

- It can be shown that in order to find $\sigma(t, S(t))$ one needs to establish the following conditional expectation:

$$\mathbb{E}[v(t)|S(t) = K].$$

- Evaluation requires joint distribution $f_{v,S}$, which is not known explicitly.
- We can obtain this expectation from the simulated Monte Carlo paths:

$$\mathbb{E}\left[v(t)|S(t) = K\right] \approx \mathbb{E}\left[v(t)|S(t) \in [b_k, b_{k+1}]\right],$$

where $K \in [b_k, b_{k+1}]$, or better:

$$\begin{aligned}\mathbb{E}\left[v(t)|S(t) = K\right] &= \lim_{\varepsilon \to 0^+} \mathbb{E}\left[v(t)|S(t) \in [K-\varepsilon, K+\varepsilon]\right] \\ &= \lim_{\varepsilon \to 0^+} \frac{\mathbb{E}\left[v(t)\mathbb{I}_{S(t)\in[K-\varepsilon,K+\varepsilon]}\right]}{\mathbb{Q}\left[S(t) \in [K-\varepsilon, K+\varepsilon]\right]}.\end{aligned}$$

- The expectations can be approximated by a proper “bundling”.
- We investigate the improvement of this technique by collocation.

The Collocation method

- Suppose we consider a problem of sampling of 1.000.000 samples from a variable Y for which the inverse CDF is not known analytically. A standard procedure is to invert numerically 1.000.000 times the CDF:

$$y_i = F_Y^{-1}(u_i), \quad u_i \sim \mathcal{U}([0,1]).$$

Problem: How to obtain 1.000.000 samples from Y by using only **a few** inversions $F_Y^{-1}(u_i)$?

The Collocation method

- The collocation technique originates from the field of Uncertainty Quantification [Babuška et al., 2007, Xiu and Hesthaven, 2005].
- We developed a technique to use the collocation method to approximate any random variable Y by a polynomial of normals (or any other variable), i.e.,

$$Y \sim a_0 + a_1 X + a_2 X^2 + a_3 X^3 + \cdots =: Z,$$

such that:

a) $\mathbb{E}[Y^n] = \mathbb{E}[Z^n], \quad \forall n \in \mathbb{N}$

b) The CDFs of Y and Z agree at the so-called collocation points.

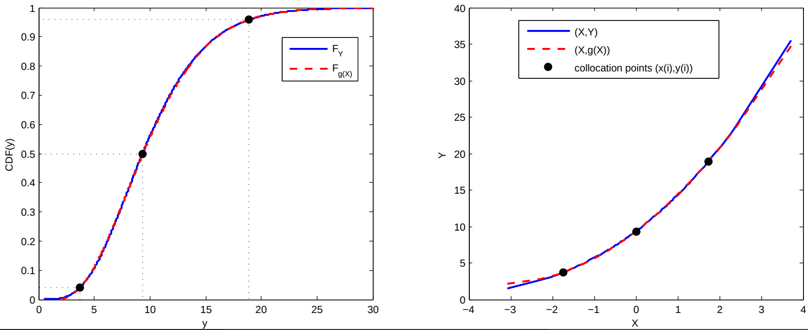
c) **No optimization technique will be used!**

The Collocation method- Example

- We consider an “expensive” variable $Y \sim \Gamma(5,2)$ and approximate if by a polynomial of standard normals:

$$g_3(X) = \sum_{i=1}^{3} F^{-1}_{\Gamma(5,2)}(F_X(x_i))\ell(X) = a + bX + cX^2.$$

- In the experiment we have generated $M = 10^6$ samples and the corresponding CDF is depicted below.

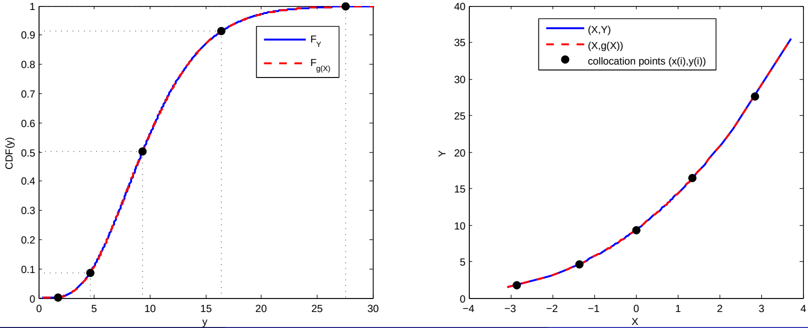


The Collocation method- Example

- We consider an "expensive" variable $Y \sim \Gamma(5,2)$ and approximate if by a polynomial of standard normals:

$$g_N(X) = \sum_{i=1}^{N} F^{-1}_{\Gamma(5,2)}(F_X(x_i))\ell(X) = a + bX + cX^2 + dX^3 + eX^4.$$

- In the experiment we have generated $M = 10^6$ samples and the corresponding CDF is depicted below.



Collocation Method and Stochastic Local Volatility Models

- Currently we investigate how to apply the collocation method for efficient evaluation of the $\mathbb{E}[v(t)|S(t)=K]$ thus how to facilitate more efficient evaluation of the stochastic-local volatility models (PhD student Anton van der Stoep).
- One of the unresolved problems in financial engineering is the "big-stepping" simulation of the SABR model.

$$\begin{aligned} \mathrm{d}S(t) &= \sigma(T)S^{\beta}(t)\mathrm{d}W_1(t), \\ \mathrm{d}\sigma(t) &= \gamma\sigma(t)\mathrm{d}W_2(t), \end{aligned}$$

  with $\mathrm{d}W_1(t)\mathrm{d}W_2(t) = \rho\mathrm{d}t$.
- With the collocation method this problem can be solved (PhD student Alvaro Leitao Rodriguez).

Fixing arbitrage in the volatility parameterizations

- When handling a large number of market quotes, it is natural to express them in parametric form so that the whole range of strikes can be explained by only a few parameters.
- The most commonly used parameterizations like Hagan's et. al or SVI lead to arbitrage.
- The arbitrage can be seen by calculating the "implied" density from the parameterization.

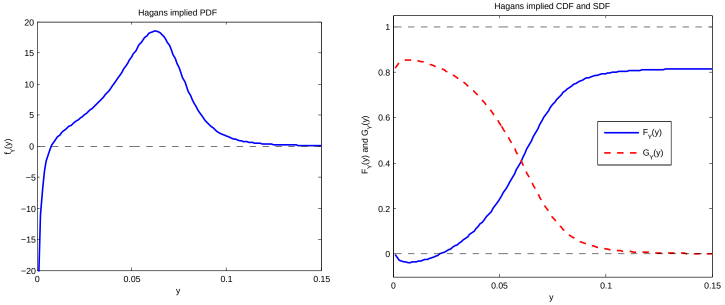
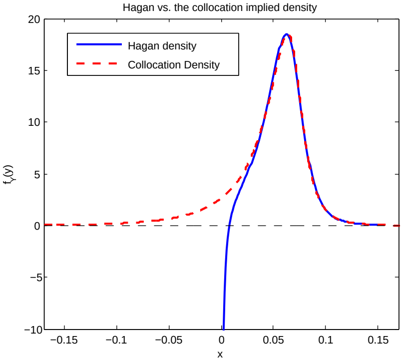
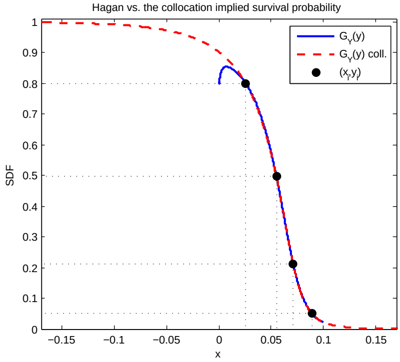


Figure: Left: probability density, with deterioration near zero; right: corresponding CDF and SDF (survival distribution function).

Fixing arbitrage in the volatility parameterizations

- By the collocation method we are able to fix the arbitrage in the parameterized volatilities.





Fixing arbitrage in the volatility parameterizations

- Currently we are able to fix the so-called "butterfly arbitrage" which can be seen as the arbitrage in the implied density.
- The remaining, unresolved, problem which is currently under investigation is to address the arbitrage in the calendar spreads (more complex as it involves transition densities).

Current & Future Research Plans

Research plans

- Explore the application of the collocation method in improving of stochastic-local volatility models for pricing Foreign-Exchange derivatives (with Anton van der Stoep).
- Inclusion of stochastic interest rates in local-volatility models.
- "Exact" and efficient simulation of the SABR model (with Alvaro Rodriguez).
- Enhance and promote usage of heavy-tailed distributions in pricing of financial derivatives.
- Improve sampling from multidimensional dependency structures (more efficient sampling from so-called trees and vines).
- Development of local-volatility Libor Market Model (highly dimensional problem) and reduce the calibration burden often present in such models.
- Fixing calendar arbitrage in the parameterized volatilities.

Long-term objective

Improve and promote models that are numerically efficient, allow for better risk management and facilitate transparent understanding of the risk factors.

Babuška, I., Nobile, F., and Tempone, R. (2007).
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Xiu, D. and Hesthaven, J. (2005).
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