WRITTEN PLANS AND THE CHALLENGES OF SELF-REGULATION IN VECTOR CALCULUS

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ABSTRACT

Vector calculus is an important branch of mathematics for many undergraduate science and engineering programmes, used, for example, in fluid dynamics, electromagnetism and control systems. Core to vector calculus are the three theorems of Green's, Stokes' and Gauss'. The rich problem-solving potential offered by these theorems makes vector calculus an ideal component within a standard calculus curriculum to bring true problem solving into the classroom. To benefit from this flexibility, we have redesigned our vector calculus curriculum to have problem solving at its core, with "decision making" as the underpinning principle. Research has shown that writing about one's problem-solving approach deepens understanding of the problem and it has been argued that expository writing engages similar cognitive processes to effective problem solving. Having observed weaknesses in self-regulatory behaviour in vector calculus problem solving, we conducted an experimental-control intervention to determine if writing a plan before attempting to solve problems with multiple solution possibilities would have any effect on problem solving behaviour. Our findings suggest that requiring students to write a plan did not generally reduce reliance on multiple strategies in problem solving and thus did not provide evidence of greater confidence. The only exception was in one programme, where the experimental group employed fewer strategies than the control group. However, planning did not improve the overall quality of problem solving: in one of the problems, students in both groups were equally likely to select an inappropriate strategy. Furthermore, the experimental group was less likely to complete the more challenging of the two problems. We consider the implications for teaching and outline directions for further investigation.

INTRODUCTION AND RATIONALE

Between the 2019–2020 and 2023–2024 academic years, the vector calculus course of the first author, at a technical university in the Netherlands, was assessed using open book, technology-mediated principles. During the exam, students had access to course notes and could use graphing software and online calculators. The emphasis of the exam shifted away from memorisation and computation in a traditional vector calculus course towards higher order thinking skills (Schoenfeld, 1985) and greater alignment with the real-world technical workplace.

In line with this shift, careful attention was given to the formulation of assessment items. To ensure high-quality assessment items, design principles were formulated, with decision making as the underpinning principle (Winne, 2015). A well-designed vector calculus problem can be solved in multiple ways, depending on the nature of the curves, surfaces and vector fields included in the problem statement. By seeking to identify characteristics of curves and vector fields and by employing the powerful theorems of vector calculus, students can choose between solution strategies of greater and lesser complexity (Craig and Akkaya, 2022; Craig, 2021).

Despite aligning the teaching principles and the resources of the course with the principle of decision making (Craig, Kula and Akkaya, 2022), teachers have observed that the students often show poor self-regulatory behaviour (Winne, 2015; Winne and Hadwin, 2013). That is, if a problem has an obvious solution strategy, for example calculating *Work* by using a line integral, students often default to that method without considering whether a simpler approach, such as applying *Green's Theorem*, might be available.

Writing explanatory strategies in the context of mathematical problem solving has been shown to be effective in encouraging deeper understanding of the mathematics involved (Craig, 2007, 2016; Pugalee, 2001), a necessary first step in effective problem solving (Pólya, 1945). In this paper we describe an initiative involving writing a plan before engaging with challenging vector calculus problems, after a class in explicit strategy instruction.

To our knowledge, use of writing a plan to aid problem solving in the context of vector calculus is novel. We add theoretical depth to our study by scrutinising our findings through the lenses of Pólya's problem-solving framework as well as a model of self-regulated learning (Winne, 2015; Winne and Hadwin, 2013).

LITERATURE REVIEW

The teaching of mathematics is multifaceted, from developing conceptual knowledge and procedural skills (Engelbrecht, Bergsten & Kågesten, 2012) to logical reasoning (Inglis & Alcock, 2012) and, key to our study, problem-solving (Schoenfeld, 1985). For engineering students in particular, problem-solving skills are essential for applying mathematics effectively to their technical disciplines. Such skills are arguably built upon foundational behaviours of self-regulated learning (SRL), such as planning, monitoring, and adapting one's approach.

To situate our study, we first clarify what is meant by "problem solving". While definitions of problem vary across the literature (Craig, 2016; Schoenfeld, 2016), we use the widely accepted definition of Schoenfeld (1985), that is:

> a task that is difficult for the individual who is trying to solve it. Moreover, that difficulty should be an *intellectual impasse* rather than a computational one…. To state things more formally, if one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem (p. 74).

This definition aligns with Pólya's (1945) seminal work, his posited stages of successful problem solving are (1) understand the problem, (2) devise a plan, (3) carry out the plan and (4) look back. Schoenfeld's definition and the problem-solving stages articulated by Pólya (1945) are comparable to similarly articulated SRL models.

We note that certain notions that are core to effective problem solving are also implicated in SRL, such as metacognition (Schoenfeld, 2016; Winne, 2015), planning (Leidinger & Perels, 2012) and decision making (Winne, 2015; Craig and Akkaya, 2022). Research on mathematical problem-solving, significantly influenced by Pólya (1945), has traditionally focused on a sequence of cognitive steps or stages. Separately, the field of SRL, as articulated by key models such as those from Zimmerman (2002) and Winne and Hadwin (2013), has focused on the cyclical, metacognitive processes learners use to manage their own learning. We suggest that the principles of SRL add nuance to studies on development of effective problem-solving abilities.

Among models of SRL, we find the Winne and Hadwin model (Winne & Hadwin, 2013) particularly useful for describing what we observe in mathematics classrooms, especially in problem solving. This model recognises four phases. In the first phase, the learner "surveys conditions that might affect work on a task" (Winne, 2015, p. 537). These conditions are both internal and external, with external conditions including objectives set for the learner. In the second phase, the learner sets goals and plans how to accomplish them. Setting these goals includes considering the “qualities of cognitive operations that might apply to the task” (p. 537) such as judging the difficulty level of those cognitive operations and how successful they have been in the past in similar situations. The third phase involves carrying out the task and the fourth concerns making adaptations for similar, future tasks. Taken together, these phases provide a metacognitive overlay to Pólya’s four stages of problem solving. Surveying conditions corresponds closely to Pólya’s emphasis on understanding the problem, while goal setting and planning can be seen as a metacognitive parallel to devising a plan. Of particular importance to the study, the role of “external conditions” in the first phase suggest that the teacher can play a role, explicitly designing a task such that certain objectives influence the goal setting in the second phase.

One such external condition is the requirement to write about one’s problem-solving process. Research has shown that writing significantly supports the “understand the problem” stage of Pólya’s framework, as the metacognitive demands of writing encourage deeper engagement and reflection (Craig, 2007, 2016; Pugalee, 2001). Importantly, this effect suggests that writing can serve as a concrete external factor to support SRL. By requiring students to articulate their goals and plans in writing, a teacher can directly influence the learner’s self-regulatory behaviour and, in turn, enhance their problem-solving ability. Apart from a few recent studies (Jones, 2020; Dray and Manogue, 2023; Khemane, Padayachee and Shaw, 2024), research on teaching and learning of vector calculus is scarce; we could not come across any that employs a SRL perspective.

Self-regulated learning (SRL) is a ubiquitous process that learners engage in constantly, not just an approach they turn on and off (Winne, 2015). As Dignath and Veenman (2021) note, “Self-regulated learners plan, monitor, and control their learning in order to reach a learning goal by enacting metacognitive strategies that support these regulation activities” (p. 590). While SRL is an ever-present aspect of both academic and everyday life, it is not an immutable trait. A key point of attention for teachers is that SRL can be supported through training (Leidinger & Perels, 2012), and this training has been found to correlate with improved academic achievement (Winne, 2015), including in specific mathematical contexts (Kistner et al., 2010). Furthermore, metacognitive self-regulation is best assessed by “online” methods — that is, during task performance — using methods like think-aloud protocols (Meijer, Veenman and van Hout-Wolters, 2006) or written explanatory strategies (Craig, 2016), which provide valuable insights for instruction and intervention (Veenman and van Cleef, 2019).

In undergraduate calculus classes, vector calculus offers a particularly rich source of genuine problems. Situations involving multiple possible solution strategies produce Schoenfeld’s “*intellectual impasse*”. To resolve such problems, students must integrate geometric information, evaluate the applicability of theorems, and make consequential decisions about which strategies are most appropriate (Craig and Akkaya, 2022). This decision-making process reflects Winne and Hadwin’s second phase of goal setting, where the learner takes into account the qualities of applicable cognitive operations and their relative difficulties. Importantly, requiring students to articulate their reasoning in writing can support this process by prompting them to clarify their analysis, goals, and plan of action. In this way, writing serves as both a metacognitive support for Pólya’s stage of understanding the problem and an external condition that strengthens SRL in vector calculus problem solving.

Building on these connections between problem solving, SRL and writing, the present study investigates students’ approaches to vector calculus problems in a classroom setting designed to compare two conditions. Specifically, the research question guiding this study was:

What similarities and differences in problem-solving behaviour can be observed between students required to write a plan before solving vector calculus problems and those who began solving immediately?

METHODOLOGY

Course Context

The vector calculus course, offered to three separate programmes, is delivered in the third quartile of the academic year, roughly February to March, with the second chance exam in April. Contact hours consist of four two-hour lectures and eight two-hour tutorials. Students attend the lectures together and then split into their respective programmes for the tutorials. Two lectures are held in the first week, one in the second and one in the third. The tutorials are spread over four weeks, with the exam on the Monday of the fifth week, as shown in Figure 1:

Figure 1: Schedule of lectures (L1-4), Master classes (A, B and C) and the exam (E)

Master Classes and Intervention Design

Three workshops, called "Master classes" were held, one for each programme. These three cohorts are hereafter referred to as programmes A, B and C. It can be seen, in Figure 1, that programmes A and B attended their Master classes shortly after the lectures ended and programme C's Master class took place just before the exam. This difference in timing affected the students' level of preparedness.

Each Master class lasted 90 minutes and was divided into two halves. The first half constituted explicit strategy instruction, in which the course lecturer gave an intensive class on decision making. Topics included clarifying what was being asked in any given problem, the nature of surfaces (closed, closable, neither), the nature of curves (closed or not) and the nature of vector fields (conservative or not). Decision trees, with which the students were already familiar, were revisited and discussed. These trees showed alternate solution strategies for problems for computing work (defined as a line integral) and flux (defined as an oriented surface integral; see Appendix). The advantages of exploiting symmetry and periodicity were demonstrated. Finally, one new method of approaching problems with divergence-free vector fields was covered, a method that had not yet been encountered in class and was new to the students.

After the first half, there was a brief break and then students were randomly divided into two groups. This division was done by means of handing out numbered cards and dividing by even and odd numbers. One group remained in the same venue and the other moved into the venue next door. The students were informed that this division was for space reasons, to simulate exam conditions. In fact, the two groups formed the experimental and control groups, which received similar, but different treatments. The incentive for students to attend the second half of the Master class was the provision of detailed and personalised feedback on their individual solution attempts.

Table 1: Attendance numbers for Master class

	A	B	C	Total
Attended first half	46	24	50	120
Stayed for second half	23	24	47	94
Of those who stayed, gave consent	22	24	47	93

Table 1 shows the number of students attending the Master class itself, that is the first half of the session. At the start of the Master class, students signed a consent form stating that data would be anonymised and that they could withdraw at any time. Only students who stayed for the second half produced any data as detailed below. In Table 2 we indicate the split within each programme into experimental and control groups.

Table 2: Split per programme into experimental and control groups

	A	B	C	Total
Experiment	13	14	24	51
Control	10	10	23	43
Total	23	24	47	84

The control group

The group that shifted to the new venue was designated the control group. In this group, once everyone was settled, each student was provided with two novel problems, each on a separate page, with space provided for writing out their solutions. They were allowed to begin solving the problems immediately.

The experimental group

The experimental group was provided with a single sheet of paper with two problems; each placed in a box with limited space provided for writing. The instruction at the top of the sheet was "*Read the provided vector calculus problems. Study any geometric objects or functions in the problem. Explain how you plan to solve the problem.*" Students were told that they would receive the solution sheets once they had written a plan and should alert a tutor when ready. In practice, some students engaged extensively in planning both questions. When the tutors observed this process to take too long, to the detriment of problem-solving time, they suggested that the student move on and gave them the handout. The time taken by individual students was not recorded.

Because the experimental group had to write a plan before attempting to solve the problems, the control group effectively had more time for problem solving. To minimise this disadvantage, the experimental group remained in the first venue so they could begin planning while the control group was still settling into the adjacent venue. Furthermore, we tried to schedule the Master classes at the end of the day so that the venue would remain available and students would not have subsequent classes. This was possible for Master classes A and C, but not for B, which was in the first two periods of the day. As far as possible, we sought to ensure that students in both groups had adequate time to work on the problems.

The problems

The two Master class problems (Figures 2 and 3) were chosen from Wade (2014) by the first author and were worked through and approved by two mathematician colleagues. The diagrams were generated in *GeoGebra*.

S is the truncated cone $y = 2\sqrt{x^2 + z^2}, 2 \leq y \leq 4$,
$\mathbf{n}$ is the outward pointing normal and
$\mathbf{F}(x, y, z) = \langle x, -2y, z \rangle$. Calculate $\iint_S \mathbf{F} \cdot \mathbf{n} dS$.

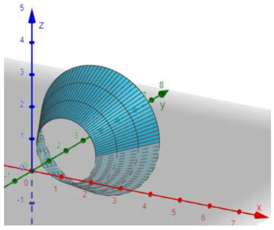

Figure 2: Problem 1

S is the portion of the plane $z = y$ which lies inside the sphere $x^2 + y^2 + z^2 = 1$, $\mathbf{n}$ is the upward pointing normal and $\mathbf{F}(x, y, z) = \langle xy, xz, -yz \rangle$.

a) Evaluate div $\mathbf{F}$. What can we conclude about $\mathbf{F}$?
b) Calculate $\iint_S \mathbf{F} \cdot \mathbf{n} dS$.

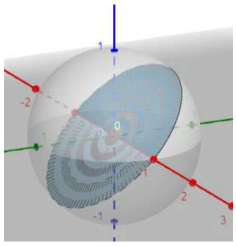

Figure 3: Problem 2

Both problems asked explicitly for a surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} dS$ to be calculated where surface *S* is provided symbolically and shown in a diagram. Students had been taught methods to calculate surface integrals, both by parametrising the surface or remaining in Cartesian form.

In both problems, the divergence of the vector field is zero. Therefore, a line integral along the boundary of *S* is a legitimate strategy, utilising the property

$$\nabla \cdot (\nabla \times \mathbf{G}) = 0$$

where we can consider **F** to be the curl of some suitable vector field **G**. **G** is not unique and can be found by inspection. For example, in Problem 1,

$$\mathbf{F} = \langle x, -2y, z \rangle = \nabla \times \langle -yz, 0, xy \rangle,$$

and in Problem 2

$$\mathbf{F} = \langle xy, xz, -yz \rangle = \nabla \times \langle \frac{1}{2} xz^2, -xyz, 0 \rangle.$$

The *Divergence Theorem*, which transforms a surface integral into a triple integral, is only applicable if a surface is closed (e.g., a sphere) or is closable. In Problem 1, the surface can be closed by introducing two discs. The surface in Problem 2 is not closed nor obviously closable. The *Divergence Theorem* is therefore an applicable strategy in Problem 1 and not in Problem 2.

Data and coding

Two forms of data were collected: written plans for the experimental group only, and written solutions of the two problems for both groups.

A coding scheme was developed for each data type and recorded in a spreadsheet. Each item was coded independently by two people, with disagreements resolved by the first author. As this paper does not report on the written plans, data and conclusions from the written plans themselves are not included and hence we shall only describe the coding of the problem solutions.

Coding of the problem solutions

Table 3 lists all recorded attributes, with those relevant to this paper italicised. Many attributes were common to both problems (e.g., number of solution strategies apparent). However, since *Divergence Theorem* was an appropriate strategy for Problem 1 but not for Problem 2, we could record progress only in the first case, but not in the second. Therefore, different data were recorded for Problem 2.

To capture students' progress, solutions were coded as follows: 0 = no progress beyond indicating a strategy, 1 = small progress, 2 = substantial progress but not completed, 3 = completed with a correct result, 4 = completed with an incorrect result, and 5 = *Divergence Theorem* as the sole strategy (Problem 2 only, as this was inapplicable). Codes 3 and 4 were classified as completion, whereas codes 0–2 and, for Problem 2, code 5 indicated non-completion.

Table 3: Recorded attributes of the problem solutions with coding detail

Attribute	Problems	Codes
Number of strategies apparent in the solution	*1 and 2*	*A number between 0 and 3*
Which strategies are apparent	1 and 2	Surface integral, line integral, Divergence Theorem, mix[a]
Surface integral	1 and 2	Presence of: normal, curl, bounds
Line integral	1 and 2	**G** calculated correctly/incorrectly [b]
Divergence Theorem	1 only	Divergence zero/nonzero Closing discs: present/not present
Divergence calculation and interpretation	2, part a	Calculation correct/incorrect Interpretation correct/incorrect/none
Divergence Theorem	*2 only*	*Surface being used specified/unspecified* *Result zero/nonzero/none*
Degree of progress (recorded separately for each appropriate strategy attempted)	*1 and 2*	*none, some (a small amount), some (a great deal), completed correctly, completed incorrectly*

[a] For the attribute "which strategies are apparent", the code "mix" refers to use of an incorrect expression which blended two strategies, for example a surface integral with div **F** as the integrand.
[b] $\boldsymbol{F} = \nabla \times \boldsymbol{G}$

Data analyses

Strategy count data, not normally distributed, were compared between groups using Mann–Whitney U tests, with descriptive statistics (means, standard deviations, totals) reported. Proportions of students selecting the *Divergence Theorem* and completing each problem were analysed using two-proportions z-tests, supplemented by Fisher's exact tests to address small, expected cell counts.

Various different statistical tests were used in this study (Mann–Whitney U, two-proportions z, and Fisher's exact tests) to explore group differences across several outcomes. The study and analysis plan were not pre-registered, and no a priori power analysis was conducted. Given the modest sample sizes, the study had limited power to detect small effects. Because multiple comparisons were performed, we recognise an increased risk of Type I error. Accordingly, p-values are reported descriptively, and the results should be interpreted with caution rather than as confirmatory evidence.

RESULTS

To address the research question "*What similarities and differences in problem-solving behaviour can be observed between students required to write a plan before solving vector calculus problems and those who began solving immediately?*" we focus on three key observations that emerged from the data, listed below.

(1) Experimental students choose fewer strategies than the control students.
(2) Experimental students were more likely than control students to choose the *Divergence Theorem* in Problem 2, even though it was not an appropriate strategy.
(3) Control students were more likely to finish problems than the experimental students.

Observation 1: Experimental students choose fewer strategies than the control students.

No overall difference was found in the number of strategies used between experimental and control students across the two problems. In Problem 2, the totals were nearly identical, with the experimental group attempting 48 strategies (M = 0.94, SD = 0.61) and the control group attempting 41 (M = 0.93, SD = 0.55). The Mann–Whitney U test confirmed this (U = 1128.0, p = .961).

Similarly, in Problem 1, the experimental group used 48 strategies versus 50 in the control group. Although the mean number of strategies was slightly lower in the experimental group (M = 0.98, SD = 0.58) than in the control group (M = 1.14, SD = 0.41), the difference was not statistically significant at the cohort level (U = 953.5, p = .089). A significant difference emerged only in Programme C (U = 228.0, p = .037), with control students attempting more strategies.

Observation 2: Experimental students were more likely to choose the Divergence Theorem in Problem 2, even though it was not an appropriate strategy.

Use of the *Divergence Theorem* is appropriate in Problem 1, but not in Problem 2. Many students nevertheless chose to apply it in Problem 2. Fourteen of 51 experimental students (27.5%) selected this strategy, compared with five of 44 control students (11.4%) (Table 4).

Table 4: number of students choosing *Divergence Theorem* in Problem 2

	Choosing DT	Group size
Experimental	14 (27.5%)	51
Control	5 (11.4%)	44

To compare the proportions, we conducted a two proportions z-test under the null hypothesis of no group difference (z = 1.99, p = .047, α =5%). Because some expected cell counts were small, we also ran Fisher's exact test (p = .071) which was not significant.

Taken together, these results suggest that experimental students were somewhat more likely than control students to choose the *Divergence Theorem* in Problem 2, although the strength of this conclusion depends on the statistical test applied.

Observation 3: Control students were more likely to complete the problems than experimental students.

Table 5 below indicates the maximum progress attained by each group for both problems. If students used multiple strategies, only the highest progress was recorded. For Problem 1, 19/44 control students (43.2%) and 18/51 experimental students (35.3%) achieved completion (codes 3–4). The difference was not statistically significant (two-proportions z = 0.71, p = .48; Fisher's exact p = .55). We therefore did not find evidence of a difference in completion between groups for Problem 1. Given the sample size, small effects may have gone undetected.

Table 5: Maximum degree of progress in problem solution

	0	1	2	3+4	5	Total
Problem 1						
Control	16	8	1	19 (43.2%)		44
Experiment	25	5	3	18 (35.3%)		51
Problem 2						
Control	18	12	2	10 (22.7%)	5	44
Experiment	24	6	6	4 (7.8%)	11[a]	51

[a] 14 students chose *Divergence Theorem* as a solution strategy, but only 11 chose it as their only solution strategy.

For Problem 2, completion was 10/44 (22.7%) in the control group and 4/51 (7.8%) in the experimental group. A two-proportions z-test showed a significant difference (z = 2.04, p= .041), confirmed by Fisher's exact test (p = .047). Thus, experimental students were less likely to finish Problem 2.

The results on completion indicate that both groups completed Problem 1 at similar rates, but more control than experimental students completed Problem 2. This suggests that requiring a plan did not increase completion and may have reduced it on more demanding tasks.

DISCUSSION

This study investigated the effect of requiring students to write a plan before solving vector calculus problems. The problems were deliberately designed to constitute an *intellectual impasse* (Schoenfeld, 1985) by allowing multiple possible solution strategies. Given the scarcity of research on vector calculus (Jones, 2020; Dray & Manogue, 2023; Khemane, Padayachee & Shaw, 2024), our findings, although limited, represent an early step in bringing a SRL perspective to this domain. Our analysis focused on three observations: (1) differences in the number of solution strategies attempted, (2) use of the *Divergence Theorem*, and (3) progress towards completion of the problems.

Across the two problems, experimental and control students used a similar number of strategies, *Observation 1* indicates that planning did not broadly reduce trial-and-error approaches. A difference emerged only in Programme C, where control students used more strategies than experimental students. As Programme C was the largest and best-prepared cohort, this may suggest that planning helps some students work more efficiently. However, since the effect did not extend to the whole cohort, it should be seen as tentative. Overall, the results point to no general impact of planning on strategy use, though programme-specific factors may shape its influence.

In Programme C, the finding of fewer strategies by experimental students can be interpreted through Pólya's (1945) model, where planning follows problem understanding, and Craig's (2007; 2016) evidence that writing deepens mathematical understanding. Within the SRL model (Winne & Hadwin, 2013), a written plan supports goal setting and monitoring, which may have enabled Programme C students to regulate their work more effectively and avoid inefficient trial-and-error (Veenman & van Cleef, 2019).

However, the value of planning cannot be judged solely on the number of solution strategies attempted. We must also consider whether writing a plan has led to better choices. In Problem 1, several approaches were equally valid. As *Observation 2* shows, in Problem 2, however, the *Divergence Theorem* was used inappropriately by 14 experimental students compared with only 5 controls. In this case, planning not only failed to improve strategy choice but may have reinforced a poorer one. As Veenman and van Cleef (2019) note, "metacognitive knowledge does not automatically lead to appropriate strategic behavior" (p. 691), a point evident in our data. Writing a plan may fulfil Winne and Hadwin's goal-setting phase, but if the task is misunderstood, the plan can consolidate that error. This underscores the importance of initial problem comprehension in both Pólya's and Winne and Hadwin's models.

The final point of interest concerns students who completed both problems, as described in *Observation 3*. Progress was coded from no attempt to full completion, with codes 3 and 4 representing completion with correct or incorrect results. In Problem 1, completion rates did not differ significantly (43% control, 35% experimental). In Problem 2, however, the experimental group completed less often (8% vs. 23%). This may reflect time constraints, since planning was required and Problem 2 was attempted second. From an SRL perspective, planning may also have made students more aware of the task's difficulty, while less thorough planning among control students could have given a false sense of manageability. Future studies could test this by reversing the order of problems.

More broadly, our results suggest that isolated interventions in planning do not necessarily improve performance. When teaching vector calculus problem solving, we model the desired behaviour such as using decision trees before attempting solutions to determine possible strategies depending on the nature of the curves, surfaces and vector fields. These are examples of indirect activation of SRL strategies. Dignath and Veenman (2021), report that explicit instruction in SRL has stronger effects than indirect activation, the Master classes provided such explicit instruction to both groups, with the experimental group experiencing the extra emphasis of being required to write a plan. However, Leidinger and Perels (2012), citing Hattie, Biggs and Purdie (1996), caution that isolated SRL instruction rarely transfers effectively. This may help explain why requiring a written plan did not consistently improve outcomes.

Future iterations of this intervention could therefore focus on embedding writing about problem solving within regular classes rather than treating it as a separate, one-off requirement. As Leidinger and Perels (2012) argue, SRL training is more effective when integrated into regular instruction. Embedding planning and reflection activities more systematically may encourage students not only to commit to strategies but also to evaluate their appropriateness and adapt when necessary.

CONCLUSION

In this paper we reported on an experiment–control study of students' self-regulatory behaviour in vector calculus problem solving, focusing on the intervention of writing a plan. Our study responds to Dignath and Veenman's (2021) call for further experimental work on explicit instruction in metacognitive strategies. Devising and recording a plan can be viewed as an external support in the goal-setting phase of SRL.

We acknowledge that our data do not provide strong or definitive evidence. Rather than firm conclusions, the study offers intriguing directions for further investigation. The finding that experimental students in one programme attempted fewer solution strategies raises the possibility that planning may help reduce trial and error, but this effect appeared only in one programme and should not be overinterpreted. The result that experimental students were more likely to adopt a strategy not valid for Problem 2 is surprising. Ideally, planning should promote more effective strategic choices, yet in this case it appeared to consolidate a poor one. Further research, perhaps incorporating student interviews or think aloud protocols (Veenman and van Cleef, 2019) is needed to clarify this result.

Devising and writing a plan for problem solving and concretising it by writing it down can be seen as an external factor in the the goal-setting phase of SRL. Our findings suggest that such an intervention may be useful in supporting confidence and reducing trial and error, but it is clearly not a complete solution.

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APPENDIX: DECISION TREE