Detecting fatigue in sports data

via time-uniform martingale bounds

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1. Introduction

- Data is obtained from runners over a long race via (inertial)-motion sensors attached to the body or force plates (embedded within shoes or treadmill).
- The goal is fatigue detection for data collected over time by studying change in the stream of the sensor data. This means segmenting data into rest and fatigue states.
- A centered martingale is constructed from the data which serves as the test statistic, with critical thresholds given by linear and LIL (law of iterated logarithm) bounds over time.
- Results: Segmentation of data into rest, fatigue states. Comparison of average stride patterns from different states leads to **region of interest (ROI)** identification in functional data.
- **Applications areas** would be any form of gait analysis in sports and medicine and change-point detection problems.

2. Data, test problem, critical threshold

- We use the *functional signal* plus *noise* model where each stride Y_i, $i = 1, ..., n$ is represented as $Y_i(t) = \mu_s(t) + \epsilon_i(t)$, with $\mu_s(t)$ as an average stride pattern for $s \in \{\text{rest, fatigue}\}$ and ϵ_i as gaussian error.
- Each of the functional curves are reduced to a single point by means of L^2-error: $\Delta_{i,s} := \|Y_i(t) - \mu_s(t)\|_2^2$ as in (3).
- For the null $H_i : \Delta_{i,s} \overset{i.i.d.}{\sim} F$, we define test problem:

$$\mathcal{H}_t = \cap_{i=1}^t H_i.$$

- With δ as level of test $\mathcal{H}_t$, consider centered martingale M_t given by, $M_t = \sum_{i=1}^t \mathbb{1}\{\Delta_{i,s} > \gamma\} - t\alpha$, where γ is local critical threshold at level-α.
- M_t (shown in (4)) serves as test statistic over time $t \in \mathbb{N}$.

Mathematical analysis of sports data: A complete cycle

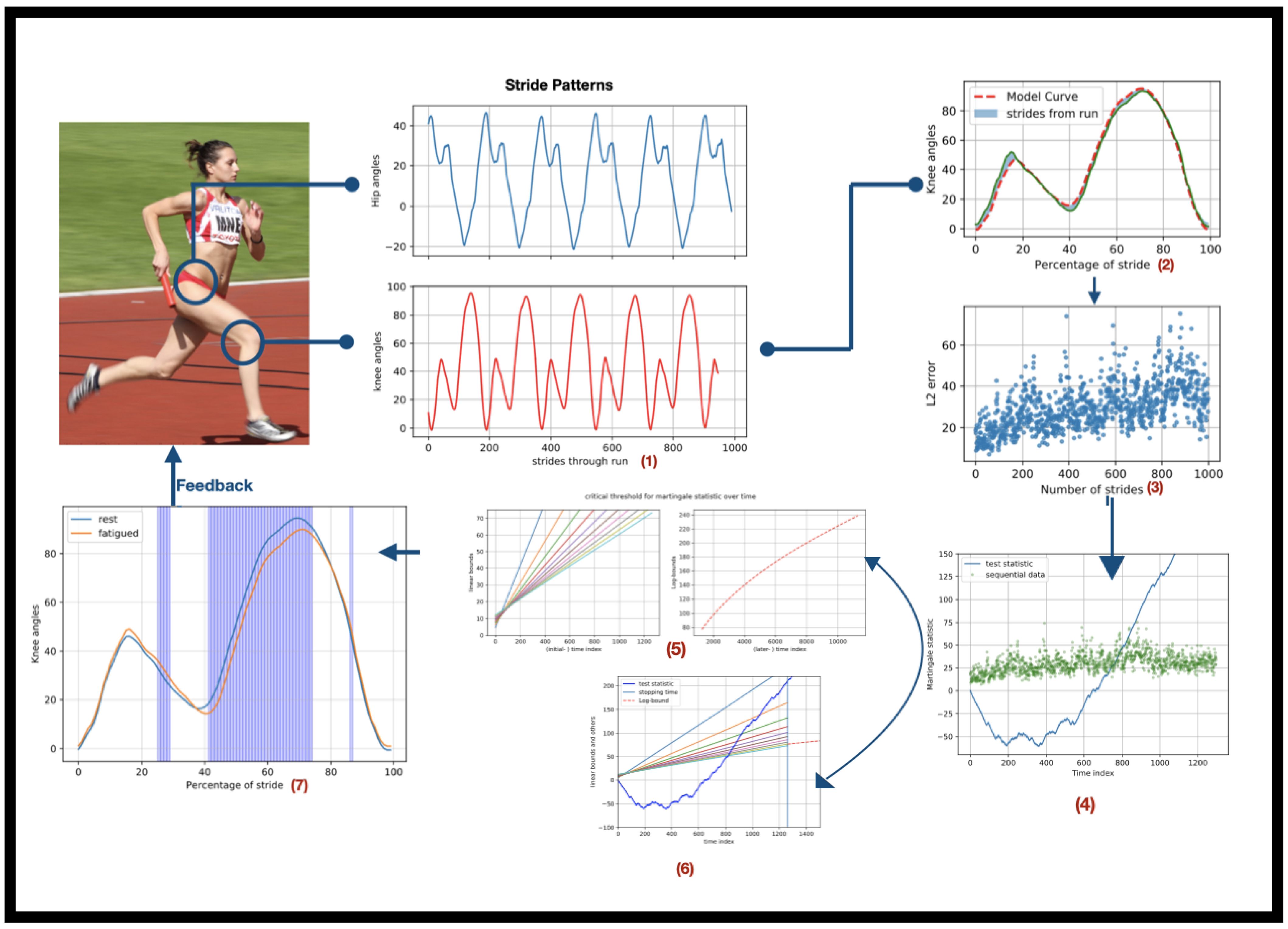

Figure 1: (top left) Runner, [wikipedia]; **(1)** Hip and Knee angles; **(2)** one stride from knee angles; **(3)** L^2-error ($\Delta_{i,s}$); **(4)** Point data over time and test statistic M_t **(5)** (left) Linear bounds $L_j(t)$ and (right) finite-LIL bound; **(6)** Upcrossing of M_t over Γ_t **(7)** ROI in knee stride and feedback

3. Critical threshold for martingale

Given $\alpha, \delta \in (0,1), p \in \mathbb{N}, t \in \mathbb{N}$ and $\tau = 1064.3 \log\left(\frac{1}{\delta}\right)$, we show that for Γ_t (threshold over time t) given by,

$$\Gamma_t = \begin{cases} L_j(t), \quad j = 1, \ldots, p; \text{ for } t < \tau \\ \min\left\{L_j(t), \sqrt{1.2t\left(2\log\log t + \log\frac{1.15}{\delta}\right)} \wedge 1 \wedge 0.04t\right\}, \ t \geq \tau \end{cases}$$

it holds for the null $\mathcal{H}_t$ with (global-) level δ that,

$$\mathbb{P}_{\mathcal{H}_t}\left(\forall t \in: \ M_t > \Gamma_t\right) \leq \delta,$$

where $L_j(t) := \frac{c_j}{\alpha(1-\alpha)t_{0j}} t + c_j$ and c_j, t_{0j} are parameters of L_j. [1, 2]

4. ROI identification and feedback

- Upcrossing of martingale over the Γ_t (in (6)) bound captures change in underlying distribution of point data in (3) while maintaining overall type-I error δ.
- Test procedure may be performed for online, sequentially arriving functional or point data and results in segmented data from various phases (or underlying distributions).
- For functional data, detecting time point of change allows for detecting further region of interest (ROI) within functions from different segments.
- **Results (for knee angle data):** ROI (in figure (7)) indicate increased knee stiffness which can lead to injuries due to reduced shock attenuation (see [3]) and therefore appropriate **feedback** has to be given to runner for improved training.

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References

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[2] Akshay Balsubramani. Sharp finite-time iterated-logarithm martingale concentration. *arXiv preprint arXiv:1405.2639*.

[3] Jasper et. al. Reenalda. Kinematics and shock attenuation during a prolonged run on the athletic track as measured with inertial magnetic measurement units. *Gait & posture*, 68:155–160.